

A GENERALIZED TRANSPORTATION MODEL IN SOCIAL ASSISTANCE MANAGEMENT

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Abstract: The most common decision methods in choosing the method of supply in social assistance management is that of auction. Just one supplier is usually the winner, but ones offer is sometimes not the optimum version. The optimum should be considered not only in financial terms but also in time terms. We solve the problem of optimally managing the food and medical care supply by means of multi-index transportation model of tri-axial type. We use the algorithm of K. B. Haley [2] for problem solving, with a model of computation elaborated in Excel. The example we give refers to the optimization of the food supply of the network of Care Centers for Elderly in Arad County.

1. INTRODUCTION

The transportation problem solves one product supply problems. Generally in social assistance, the supply problems refer to more products as food, drugs, clothes, etc. Therefore the classical transportation problem is not enough. A multi-index transportation model is suitable in this framework. We deal with tri-axial problem [3], which contains three types of linear constraints.

The standard form of the model is, according to [1]:

$$f = \sum_i \sum_j \sum_k c_{ijk} x_{ijk} \rightarrow \min, \quad (1)$$

submitted to:
$$\sum_{k=1}^p x_{ijk} = a_{ij}, \quad i \in I, \quad j \in J, \quad (2)$$

$$\sum_{i=1}^m x_{ijk} = b_{jk}, \quad j \in J, \quad k \in K, \quad (3)$$

$$\sum_{j=1}^n x_{ijk} = c_{ik}, \quad i \in I, \quad k \in K, \quad (4)$$

$$x_{ijk} \geq 0, \quad i \in I, \quad j \in J, \quad k \in K \quad (5)$$

The terms on the right side should satisfy the conditions:

$$\sum_j a_{ij} = \sum_k c_{ik} = a_i, \quad i \in I, \quad (6)$$

$$\sum_{k=1}^p x_{ijk} = a_{ij}, \quad i \in I, \quad j \in J, \quad (7)$$

$$\sum_j b_{jk} = \sum_i c_{ik} = c_k, \quad k \in K, \quad (8)$$

$$\sum_i a_i = \sum_j b_j = \sum_k c_k = S, \quad (9)$$

$$c_{ijk} \geq 0, \quad a_{ij} > 0, \quad b_{jk} > 0, \quad c_{ik} > 0, \quad i \in I, \quad j \in J, \quad k \in K.$$

The following existence and optimality results have been proven in [2].

Theorem 1. *If $m, n, p > 1$, then the number of the linear independent equations from the constraints of the model does never exceed the number of unknowns.*

Theorem 2. *The tri-axial problem has an infinity of admissible solutions.*

Theorem 3. *The tri-axial problem has at least an optimal solution.*

Remark 4. The tri-axial transportation problem is non-degenerated if any program contains exactly $M = mnp - (m - 1)(n - 1)(p - 1)$ values $x_{ijk} > 0$, the other ones being 0. If a program contains $M - r$ values $x_{ijk} > 0$ ($r \in \mathbb{N} \geq 1$) the problem is degenerated, having a multi-plane degeneration of order r .

In order to determine an optimal program of a tri-axial transportation problem we use the Haley [2] procedure:

Step a. Determine a scale repartition using the recurrence formula

$$u_{ij} + v_{jk} + w_{ik} = c_{ijk}, \quad (10)$$

which is admissible if the non-negativity conditions:

$$x_{ijk}^{\circ} = \min(a_{ij} - \sum_{k=1}^{k-1} x_{ijk}^{\circ}, b_{jk} - \sum_{i=1}^{i-1} x_{ijk}^{\circ}, c_{ik} - \sum_{j=1}^{j-1} x_{ijk}^{\circ}) \geq 0, \quad i \in I, j \in J, k \in K.$$

are fulfilled.

Step b. Verify the optimality of the solution extending the modified distributive method (see [1]) to this type of problems. To do that one solves the system (10), c_{ijk} being the costs corresponding to the values $x_{ijk} > 0$. As in case of a non-degenerate solution there are exactly

$$M = mn + np + mp - (m + n + p - 1)$$

values $x_{ijk} > 0$, and the system (10) has $mn + np + mp$ unknowns, one consider arbitrary $m + n + p - 1$ unknowns equal to 0. So, the system (10) will be compatible, with an unique solution. Then, one determines the costs

$$\bar{c}_{ijk} = u_{ij} + v_{jk} + w_{ik} \quad (11)$$

corresponding to values $x_{ijk} = 0$.

The following situations are possible:

- If: $c_{ijk} > \bar{c}_{ijk}$, for $x_{ijk} = 0$,
 $c_{ijk} = \bar{c}_{ijk}$, for $x_{ijk} > 0$,
then the solution is optimal and unique.
- If $c_{ijk} > \bar{c}_{ijk}$, for $x_{ijk} = 0$,
 $c_{ijk} = \bar{c}_{ijk}$, for some $x_{ijk} = 0$,
then the problem has an infinity of optimal solutions.
- If $c_{ijk} < \bar{c}_{ijk}$, for some $x_{ijk} = 0$
then the solution is not optimal and may be improved.

Step c. Improve the solution for each cycle corresponding to the cells in which

$$c_{ijk} < \bar{c}_{ijk}.$$

The optimality is verified after improving each cycle, which leads to huge calculus if there are numerous cells in which $c_{ijk} < \bar{c}_{ijk}$.

2. APPLICATION IN SOCIAL ASSISTANCE MANAGEMENT

We use the above described multi-index transportation problem for solving the problem of optimizing the process of food supply in case of the Care Centers for Elderly on Arad County area. In the framework of the Social Assistance Direction of Arad county, there are three subordinate budgetary centers: Center 1, Center 2 and Center 3. The Direction has to ensure the delivery of the necessary goods (foods, medicines, materials) to the subordinate centers.

The problem of delivery efficiency has three parts: ensuring the need of goods, reducing the delivery time and reducing the delivery costs. This article will solve the problem of ensuring the need of goods and reducing the delivery costs suggesting a shaping as a multidimensional transport issue. The suggested algorithm takes into consideration the upgrade of delivery regularity, minimizing the cost for each delivery trance fulfilled.

Next we are going to use the following notations:

$i \in I$ - denomination given to the center of production / providers

$j \in J$ - denomination given to the product (or to the assortment which is about to be sent)

$k \in K$ - denomination given to the consumer center which has to get the product (or the assortment)

a_{ij} - denomination given to the quantity of product j , available in the production center i ;

b_{jk} - denomination given to the quantity of product j , necessary to the consumer center k ;

c_{ik} - denomination given to the quantity of products which are to be sent from the producing center i to the consumer center k ;

a_i - denomination given to the whole quantity of those n products available in the production center i ;

b_j - designation given to the quantity of product j available in the production centers m ;

c_k - designation given to the whole quantity of that n product necessary to the consumer center k ;

c_{ijk} - designation given to the delivery cost for one piece of the product j , produced in the production center i and sent to the consumer center k ;

x_{ijk}° - designation given to the quantity of product j which is to be sent from the production center i to the consumer center k .

We are going to make an algorithm for each class of products mainly for food. For food supply have the following data:

- Three caterers:

Name for the production center	Denomination
Caterer A	i=1
Caterer B	i=2
Caterer C	i=3

- Ten categories of products:

Product name	Denomination
Dairy products	j=1
Poultry	j=2
Pork	j=3
Beef	j=4
Breeding products and pasta	j=5
Vegetables	j=6
Fruit	j=7
Soft drinks	j=8
Sweets and deserts	j=9
Cans	j=10

- Three consumer centers, Care Centers for Elderly of Arad County:

Consumer center	Denomination
Center 1	k=1
Center 2	k=2
Center 3	k=3

To determine a proper transport schedule we use the data from Table 1.

Step a. The determination of the ininitial solution:

Using the recurrence formula

$$x_{ijk}^{\circ} = \min(a_{ij} - \sum_{k=1}^{k-1} x_{ijk}^{\circ}; b_{jk} - \sum_{i=1}^{i-1} x_{ijk}^{\circ}; c_{ik} - \sum_{j=1}^{j-1} x_{ijk}^{\circ}) \geq 0$$

we succesively get:

$$x_{111}^{\circ} = \min(a_{11}; b_{11}; c_{11}) = \min(69, 23, 159) = 23$$

.....

$$x_{3103}^{\circ} = \min(a_{310} - \sum_{k=1}^{k-1} x_{310k}^{\circ}; b_{103} - \sum_{i=1}^{i-1} x_{i103}^{\circ}; c_{33} - \sum_{j=1}^{j-1} x_{3j3}^{\circ}) = \min(12, 12, 10) = 10$$

initial solution thus obtained in Table 2 leads us to the admissible solution.

10		5		13		7		6		11		15		17		6		7		169		
15		17		15		8		3		8		6		9		10		9		169		
	11		20		16		9		2		7		9		8		9		11		151	
		69		59		57		52		47		45		47		61		38		14		498
12		19		20		19		23		19		5		18		21		18		169		
	11		12		17		11		17		8		20		9		15		12		159	
	9		16		27		9		15		5		13		13		17		6		167	
		45		59		57		43		39		25		43		47		67		70		495
6		6		15		20		29		17		19		9		21		14		178		
	4		3		7		5		17		15		12		17		13		9		150	
	14		10		19		13		15		13		10		5		9		24		142	
		10		18		12		18		74		69		70		77		69		53		470
23		53		42		50		55		69		53		67		185		63		41		516
	66		43		41		26		63		19		59		59		67		35		478	
	35		40		43		37		42		51		48		59		44		61		460	
		124		136		126		113		160		139		160		185		174		137		1454

Table 1. Initial data

This solution is not degenerated because it exactly contains:

$$M = mnp - (m - 1)(n - 1)(p - 1) = 90 - 36 = 54$$

values $x_{ijk}^{\circ} > 0$, the other $mnp - M = 90 - 54 = 36$ values equal to 0.

Step b. Checking optimality:

We rewrite the costs c_{ijk} that correspond to the values $x_{ijk}^{\circ} > 0$ from the Table 1 into the Table 3 and solve the system:

$$u_{ij} + v_{jk} + w_{ik} = c_{ijk}; \quad i = \overline{1,3}; \quad j = \overline{1,10}; \quad k = \overline{1,3}.$$

As this system contains $mnp = 90$ unknowns and only $M = 54$ equations, as we considered in an arbitrary way:

$$u_{12} = 0, u_{13} = 0, u_{14} = 0, u_{15} = 0, u_{16} = 0, u_{17} = 0, u_{18} = 0, u_{19} = 0, u_{110} = 0;$$

$$u_{21} = 0, u_{22} = 0, u_{23} = 0, u_{24} = 0, u_{25} = 0,$$

$$u_{26} = 0, u_{27} = 0, u_{28} = 0, u_{29} = 0, u_{210} = 0;$$

$$w_{11} = 0, w_{13} = 0, w_{21} = 0, w_{22} = 0, w_{23} = 0, w_{31} = 0, w_{33} = 0.$$

The other values u_{ij}, v_{jk}, w_{ik} are directly obtained in Table 3. Using the relation

$$\bar{c}_{ijk} = u_{ij} + v_{jk} + w_{ik}$$

we can also determine the costs \bar{c}_{ijk} that correspond to the 36 values $x_{ijk}^{\circ} = 0$.

10		5		13		7		8		11	0		15	0		17	0		8	0		7	0		0													
15		17		15		8		3		8		8		8		9	-8		10	9		9	8		-8													
	10	15		20	0	16	0	9	0	2	0	7		9		8		9		9		11		0														
		10		0		0		0		0		0		0		0		0		0		0		0														
12	0		19	5		20	13		19	7		23		19		5		18		21		18	0	0														
11		12		17		11		17	0		7	0		20	0		9	0		15		12		0														
	9		16		27		9		16		8	5		13		13	3		17	0		8		0														
		0		0		0		0		0		0		0		0		0		0		0		0														
8	0		8	5		16	13		20	7		29		17		19		9		21		14		0														
4	0		3	1		7	8		5	0		17		15	-11		12		17		13		35	-11														
	14		10		19		13		16		13		10	9		5		9		9		24		0														
		0		0		0		0		0		0		0		0		0		0		0		0														
0	11	9		5	12	0		13	17	0		7	11	0		8	0	0		0	0	7		0	0	9		0	0	8		0	15	0		0	12	0

Table 4

Modifying this cycle we obtain a better solution in Table 5 .

10		5		13		7		8		11	0		15	0		17	0		8	0		7	0		0													
15		17		15		8		3		8		8		8		9	-8		10	9		9	8		-8													
	25	15		20	0	16	0	9	0	2	0	7		9		8		9		9		11		0														
		10		0		0		0		0		0		0		0		0		0		0		0														
12	0		19	5		20	13		19	7		23		19		5		18		21		18	0	0														
11		12		17		11		17	0		7	0		20	0		9	0		15		12		0														
	9		16		27		9		16		8	5		13		13	3		17	0		8		0														
		0		0		0		0		0		0		0		0		0		0		0		0														
8	0		8	5		15	13		20	7		29		17		19		9		21		14		0														
4	0		3	1		7	8		5	0		17		15	-11		12		17		13		35	-11														
	14		10		19		13		16		13		10	9		5		9		9		24		0														
		0		0		0		0		0		0		0		0		0		0		0		0														
0	11	9		5	12	0		13	17	0		7	11	0		8	0	0		0	0	7		0	0	9		0	0	8		0	15	0		0	12	0

Table 5.

Checking the optimality of this solution se observe that it is proper because $c_{ijk} > \bar{c}_{ijk}$ for $x_{ijk} = 0$.

Problem T-3A admits an infinity of proper programs (minimums), but all these programs lead to the same minimum value of a: $\min f = 1737$.

The interpretation of the results from Tabel 5 is done like: $x_{111}^{\circ} = 10$ represents the quantity of diary products which is to be sent from Caterer A to Center 1, etc. The same procedure is applied for optimizing the delivery time.

The use of multi-index transportation model is more suitable than the linear programming, together with the related tool pack from Excel. The multi-index transportation problem is equivalent to a linear programming problem of enormous dimension. It is difficult to approach due to the large simplex table, which should be written, the most values being zero. Also, the volume of calculus involved considerably reduces when multi-index transportation model is used.

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