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HIGHLIGHTING THE INFLUENCE OF GEOMETRICAL PARAMETERS ON THE EFFICIENCY OF THE SPUR GEAR

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Abstract: The paper highlights the way the geometrical parameters of the gear influence its efficiency. This aspect enables the optimisation of the design calculation in view of obtaining a highly-efficient gear. The following parameters are highlighted: the number of teeth of the pinion, the module, the gear ratio, the spur addendum, the friction coefficient, the mesh angle, the teeth correction.

1. INTRODUCTION

The efficiency of machines and mechanisms is an important criterion that their design is based on. In very many cases, the efficiency is the one that sells a product better on the market to the disadvantage of another one with high energy consumption. Gears are highly efficient. The efficiency of the spur gear records values within the 0.98 to 0.99 range. The improvement of the gear performance is not significantly important for a technological machine with a relatively low required power. The energy loss is considerable for a technological machine with high power consumption, which also includes a gear train with many pairs of gears. For this purpose, based on the relations set forth in the specialised literature [1] [4], the hereby paper intends to highlight the way efficiency is influenced by the different geometrical parameters of the gear, as well as by the friction coefficient.

There was considered the way the efficiency of the gear depends on the size of the module, the addendum of the spurs, the mesh angle, the friction coefficient, the number of teeth of the pinion, the teeth correction and the gear ratio. There shall be taken the case wherein the contact ratio is comprised within the $1 < \varepsilon \leq 2$ interval, i.e. maximum two pairs of spurs are simultaneously used in the gear.

2. CALCULUS RELATIONSHIPS

During the functioning of a gear, an average and an instantaneous efficiency can be defined. The instantaneous efficiency is extremely important for some mechanisms, like the cam mechanisms. A low instantaneous efficiency, at certain moments of their functioning can lead to the blockage of the mechanism. For gears, efficiency does not vary significantly and thus it does not require careful studying. However, the term of instantaneous efficiency is used in order to obtain, by its integration in a base pitch, the average efficiency.

The average efficiency can be defined as follows:

$$\eta_i = \frac{P_{ui}}{P_m} = 1 - \frac{P_{fi}}{P_m} \tag{1}$$

where:

P_{ui} is the instantaneous useful work, calculated as the difference $P_{ui} = P_m - P_{fi}$. It is the power required for the achievement of the useful work by the technological machine;

P_{fi} – power by friction from the kinematics coupling, upon coupling of the kinematic elements in motion and the environment they work in; in our study, the loss of power upon coupling of the spurs in the gear is considered;

P_m – the power, which the power entering the technological machine; in our case, it is the power generated by the driver gear.

The instantaneous power lost by friction can be defined as the product between the friction force μF_n and the relative sliding velocity of the teeth flanks, v_r:

$$P_{fi} = F_{fi} \cdot v_r = \mu F_n \cdot v_r \quad (2)$$

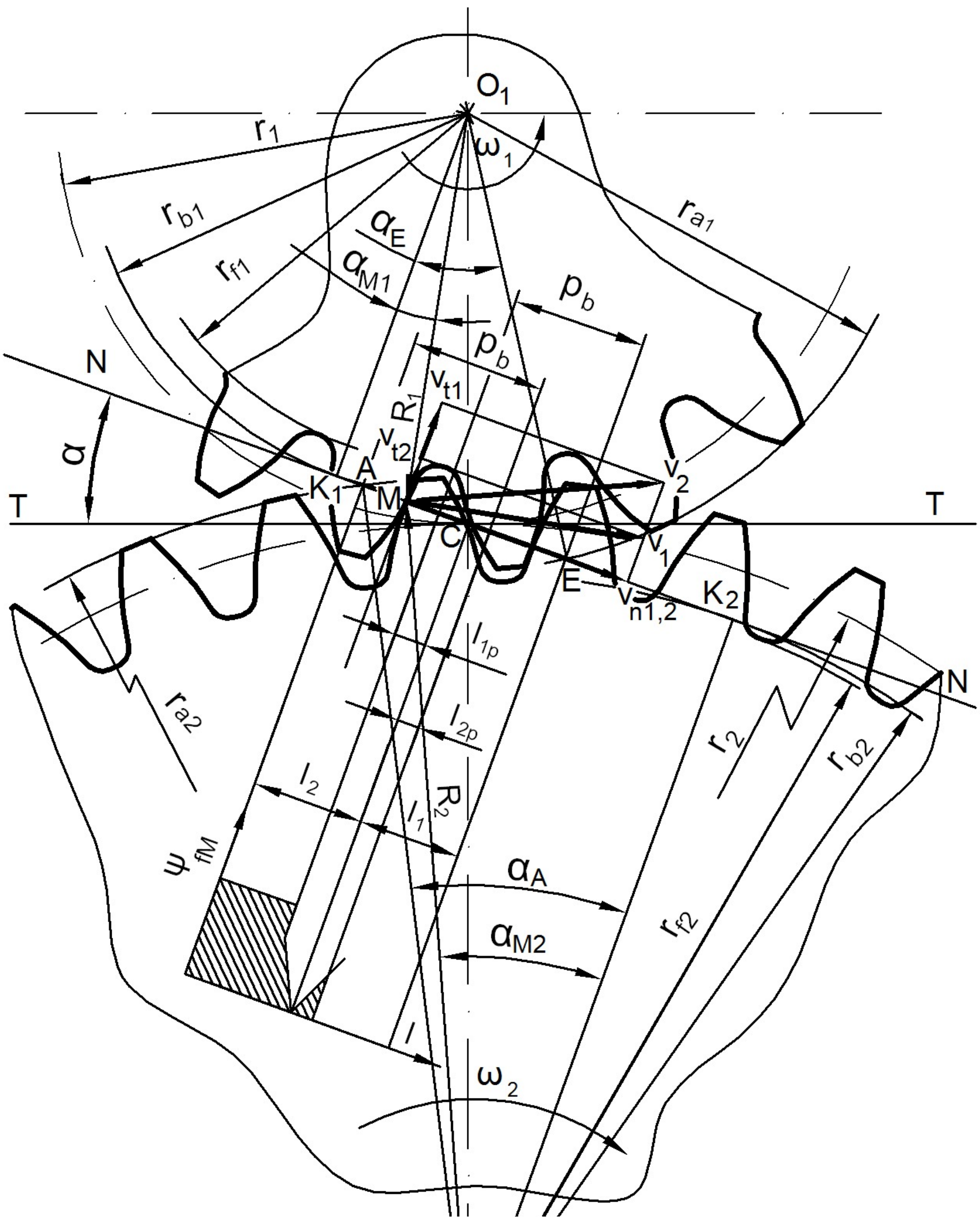

Fig.1. Geometrical and kinetic elements of the gear

The power P_m is defined as the product between the normal force F_n and the normal velocity of the coupled flank, or, so as to have a simpler calculation, as the product between the torque M_{t1} and the angular velocity ω_1:

$$P_m = F_n \cdot v_n \text{ or } P_m = M_{t1} \cdot \omega_1 \quad (3)$$

The average efficiency is defined as the ratio between the useful work and the mechanical work:

$$\eta = \frac{L_u}{L_m} = 1 - \frac{L_f}{L_m}$$

Whereof:

L_f is the mechanical work lost by friction in a kinematic cycle;
L_m is the work done.
In order to set up the expression of the efficiency of the spur gear, Fig.1 is considered with the gear specific symbols. M is a current point of the spurs in the gear:

$$v_1 = R_1 \cdot \omega_1 ;\ v_2 = R_2 \cdot \omega_2$$
$$v_{n1} = v_1 \cdot \cos\alpha_{M1} ; v_{n2} = v_2 \cdot \cos\alpha_{M2} ; v_{n1} = v_{n2}$$
$$v_{t1} = v_1 \cdot \sin\alpha_{M1} = R_1 \cdot \omega_1 \cdot \sin\alpha_{M1} ; v_{t2} = v_2 \cdot \sin\alpha_{M2} = R_2 \cdot \omega_2 \cdot \sin\alpha_{M2}$$
$$v_r = v_{t2} - v_{t1} = R_2 \cdot \omega_2 \cdot \sin\alpha_{M2} - R_1 \cdot \omega_1 \cdot \sin\alpha_{M1} = \overline{K_2M} \cdot \omega_2 - \overline{K_1M} \cdot \omega_1 \quad (4)$$
$$v_r = \overline{K_2M} \cdot \omega_2 - \overline{K_1M} \cdot \omega_1 = \omega_2(\overline{K_2C} + \overline{CM}) - \omega_1(\overline{K_1C} + \overline{CM}) =$$
$$(\omega_2 + \omega_1) \cdot \overline{CM} + (\overline{K_2C} \cdot \omega_2 - \overline{K_1C} \cdot \omega_1) = \overline{CM}(\omega_2 + \omega_1) = l \cdot (\omega_2 + \omega_1).$$

The power lost by friction is:

$$P_f = \mu F_n \cdot v_r = \mu F_n \cdot l \cdot (\omega_2 + \omega_1)$$
$$P_{f\,max} = \mu F_n \cdot \overline{AC} \cdot (\omega_2 + \omega_1) \text{ or } P_{f\,max} = \mu F_n \cdot \overline{EC} \cdot (\omega_2 + \omega_1) \quad (5)$$

(The relative sliding is null in the pitch point and maximum in the gear approach point A or the gear recess point E).
We consider that the normal force is equally allocated to the two pairs of spurs in the gear and having in view that $\overline{AE} = p_b$, then:

$$P_{f\,max} = \frac{1}{2}\mu F_n \cdot (\omega_2 + \omega_1) \cdot p_b \quad (6)$$

The power upon approaching the gear is:

$$P_m = P_u + P_f = F_n \cdot r_{b1} \cdot \omega_1 + \frac{1}{2} F_n \cdot \mu \cdot p_b \cdot \omega_1 = F_n \cdot \omega_1 (r_{b1} + \frac{1}{2}\mu \cdot p_b) \quad (7)$$

The specific loss in the gear, corresponding to point M, is the following [4]:

$$\Psi_{fM} = \frac{P_f}{P_m} = \frac{\mu\pi}{1 + \mu\frac{\pi}{z_1}}(\frac{1}{z_1} + \frac{1}{z_2}) \quad (8)$$

Where, the following were substituted: $p_b = p \cdot \cos\alpha$; $|\omega_1 / \omega_2| = z_2 / z_1$; $r_{b1} = r_1 \cdot \cos\alpha$.
Calculating the specific average loss by friction Ψ_f , from the approaching of a pair of spurs to the gear up to its recession from the gear, the following is obtained [4]:

$$\Psi_f = \frac{\int_0^{p_b} \Psi_{fM} \cdot dl}{p_b}$$

or, considering Fig. 1,

$$\Psi_f = \mu \frac{p_b(l_2 - l_{1p}) + l_{1p}^2 + l_{2p}^2}{p_b} \cdot \frac{1}{m \cdot \cos\alpha}(\frac{1}{z_1} + \frac{1}{z_2}) \cdot \frac{1}{1 + \mu\frac{\pi}{z_1}} \quad (9)$$

where $l_2 = r_{b2}(tg\alpha_A - tg\alpha_w)$; $l_1 = r_{b1}(tg\alpha_E - tg\alpha_w)$; $l_{2p} = p_b - l_1$; $l_{1p} = p_b - l_2$.
The efficiency is $\eta = 1 - \Psi_f$. (10)
1. Influence of the tooth module on efficiency

$$tg\alpha_A = \sqrt{\left(\frac{r_{a2}}{r_{b2}}\right)^2 - 1}\,; \qquad tg\alpha_A = \sqrt{\left(\frac{r_{a2}}{r_{b2}}\right)^2 - 1}$$

$$l_2 = r_2 \cos\alpha \cdot \left(\sqrt{\left(\frac{r_{a2}}{r_{b2}}\right)^2 - 1} - tg\alpha_w\right) = \frac{m \cdot z_2}{2}\cos\alpha\left(\sqrt{\left(\frac{z_2 + 2h^*_{a2} + 2x_2}{z_2 \cdot \cos\alpha}\right) - 1} - tg\alpha_w\right) = \frac{z_2}{2\pi} p_b \cdot e_2$$

Where $$e_2 = \sqrt{\left(\frac{z_2 + 2h^*_{a2} + 2x_2}{z_2 \cdot \cos\alpha}\right) - 1} - tg\alpha_w \qquad (11)$$

Similarly, the following are calculated:

$$l_1 = \frac{z_1}{2\pi} p_b \cdot e_1\,; \quad l_{2p} = p_b - l_2 = \frac{p_b}{2\pi}(2\pi - e_2 z_2);\ l_{1p} = p_b - l_1 = \frac{p_b}{2\pi}(2\pi - e_1 z_1) \qquad (12)$$

and $$p_b(l_2 - l_{1p}) = \frac{p_b^2}{2\pi}(z_1 e_1 + z_2 e_2 - 2\pi) \qquad (13)$$

Substituting the above calculations in the specific average loss, then:

$$\Psi_f = \mu \frac{2\pi(z_1 e_1 + z_2 e_2 - 2\pi) + (2\pi - e_1 z_1)^2 + (2\pi - e_2 z_2)^2}{4\left(1 + \mu \frac{\pi}{z_1}\right)}\left(\frac{1}{z_1} + \frac{1}{z_2}\right) \qquad (14)$$

From the result (14), there can be noticed that efficiency does not depend on the module and therefore it is an important aspect that needs to be considered upon the design of a gear. It is know that the centre distance results from the pre-sizing calculation and afterwards the module is calculated according to the following formula:

$$m = \frac{2a_w}{z_1 + z_2}\,. \qquad (15)$$

A smaller module and a higher number of spurs for the pinion can be chosen. As it can be seen in the Fig.2, the efficiency is better when z_1 is higher.

For a particular case where: $z_1 = 20$ teeth, $h^*_{a1} = h^*_{a2} = 1$, m=3,5, u=3, $\mu = 0{,}08$, $\alpha_w = \alpha = 20^0$, x_1=x_2=0, the efficiency is η=98,79%.

3. EFFICIENCY VARIATION DEPENDING ON DIFFERENT FACTORS (GRAPHICS)

The variation of efficiency according to the friction coefficient and the geometrical parameters is shown in Figures 2…11. The elements used in the previous example are applied in all graphics, except the varying units, one or two units that play the role of the variables.

4.CONCLUSIONS

The relation (14) established that the efficiency does not depend on module. The Fig.2…11 prove that the efficiency has high values for mesh angle $\alpha_w = 22^0 30'$, gear ratio u=5, the number of teeth of driver gear $z_1 > 30$, negative correcting teeth factor.

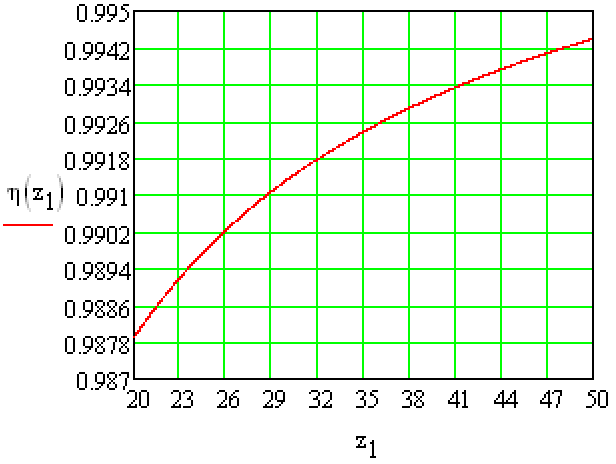

Fig.2. The dependence on teeh number

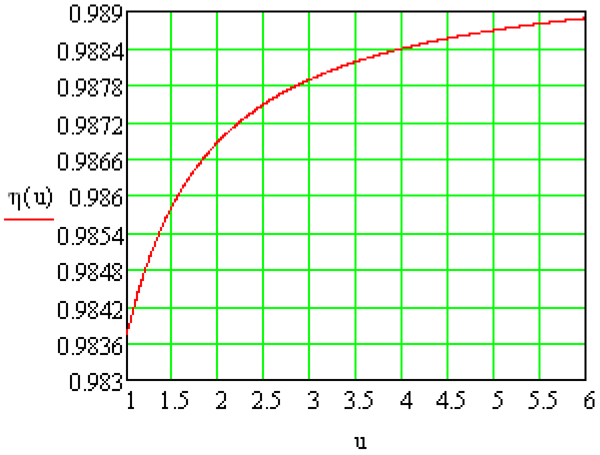

Fig.3. The dependence on gear ratio

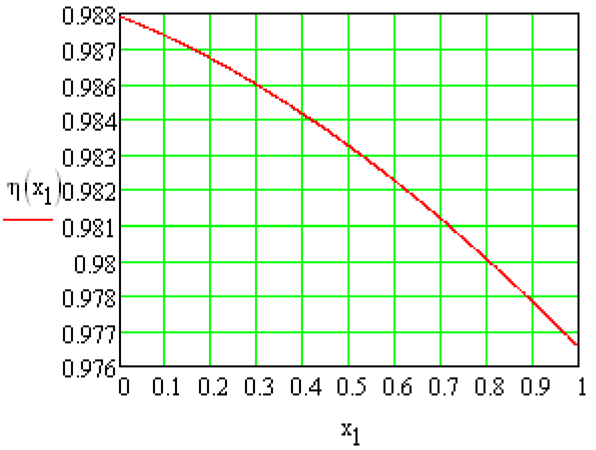

Fig.4. The dependence on correcting teeh factor for the driver gear

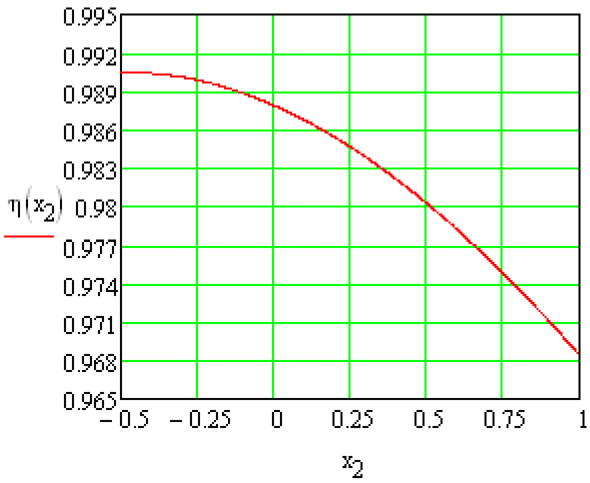

Fig.5. The dependence on correcting teeh factor for the driven gear

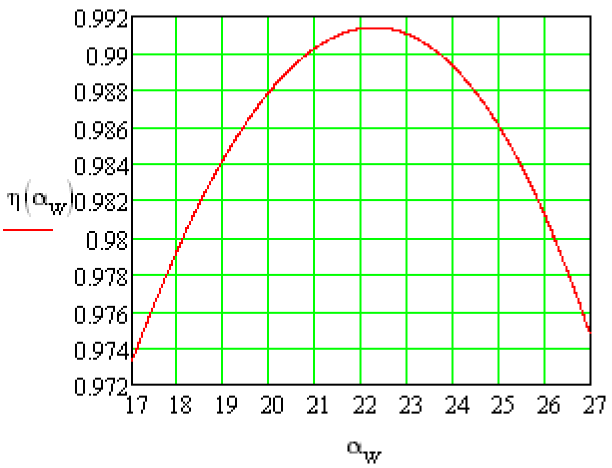

Fig.6. The dependence on mesh angle

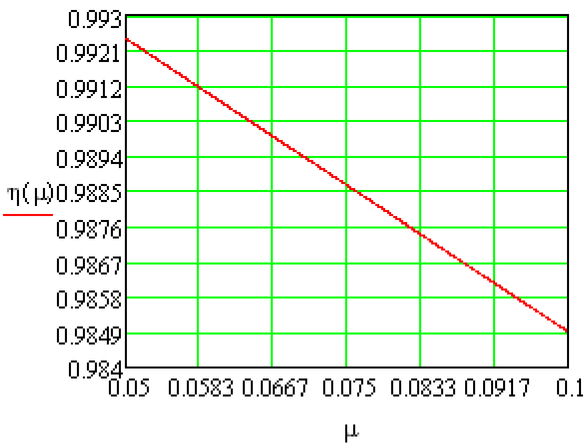

Fig.7. The dependence on friction coefficient

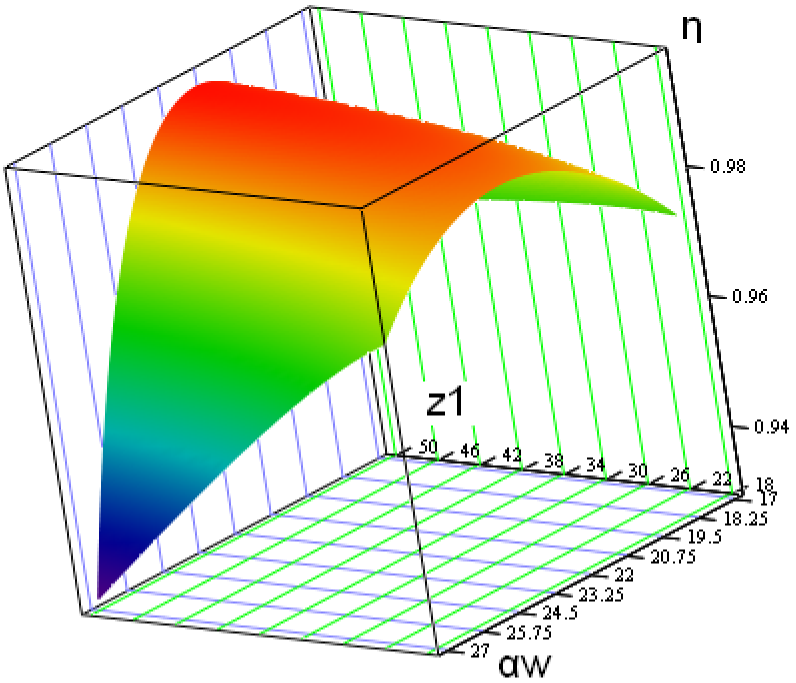

Fig.8. Efficiency variation depending on the number of teeth of pinion and mesh angle

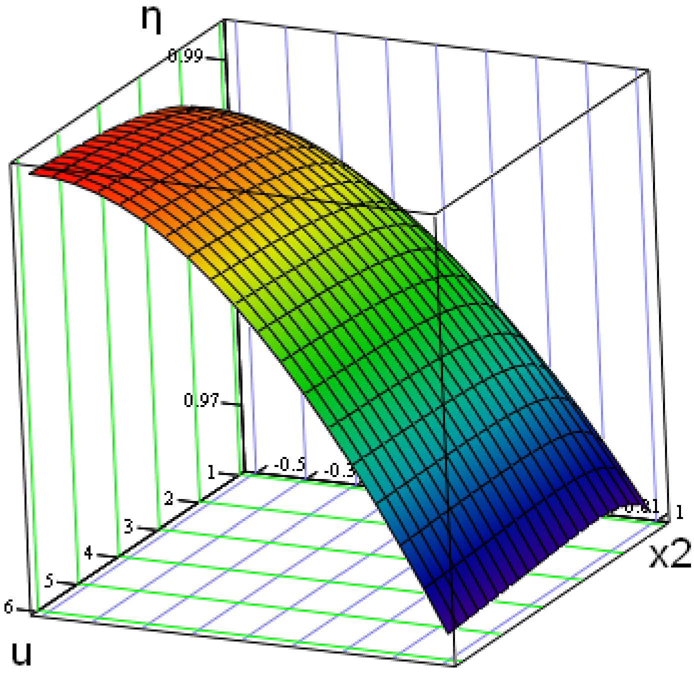

Fig.9. Efficiency variation depending on the correcting teeth factor of gear and gear ratio

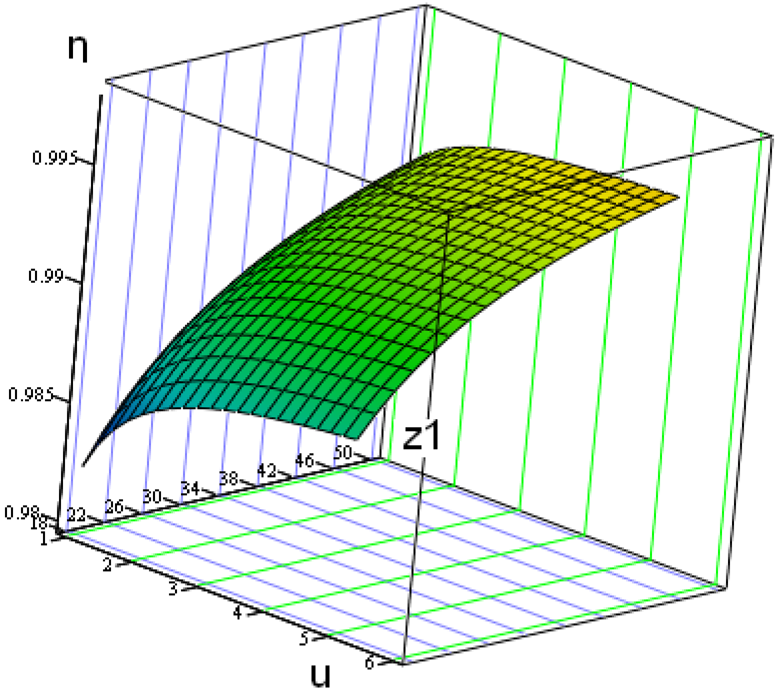

Fig.10. Efficiency variation depending on the number of teeth of pinion and gear ratio

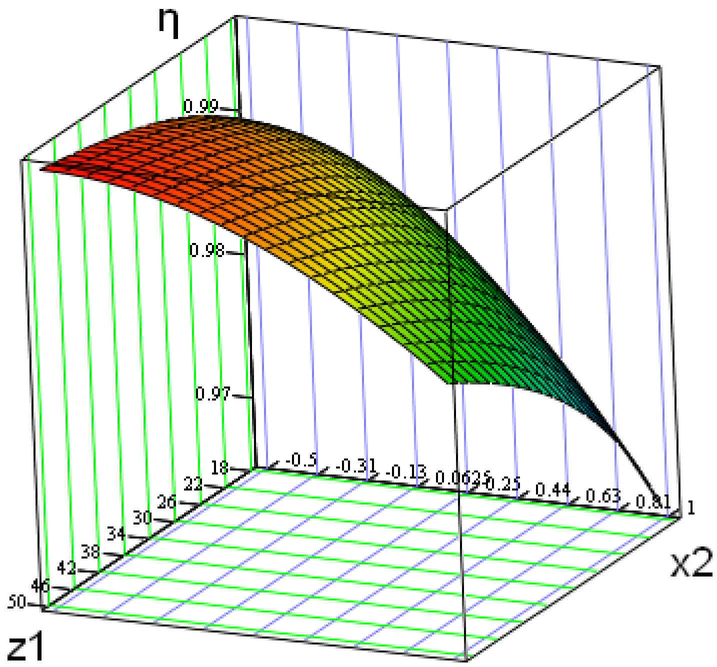

Fig.11. Efficiency variation depending on the correcting teeth factor of gear and the teeth number of pinion

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