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NUMERICAL SIMULATION OF THE ACOUSTIC WAVES PROPAGATION IN A STANDING WAVE TUBE WITH A SOUND ABSORBENT MATERIAL USING THE FINITE ELEMENT METHOD.

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ABSTRACT There is an increase in studies that try to obtain mathematical models to predict the acoustical behaviour of some sound absorbent materials, as well as those that are looking for new materials to improve the acoustic insulation and acoustic conditioning. In this work, the numerical simulation technique is applied to study the propagation of the plane acoustic waves inside a standing wave tube by means of the application of a harmonic analysis, studying the distribution of pressures. Comparing the numerical results with the experimental ones, the validity of the experimental method based on the transfer function is evaluated.

1. INTRODUCTION

The method based on the standing wave tube (or Kundt tube) for measuring the sound absorption coefficient in porous and fibrous materials is one of the most important techniques in the acoustic materials characterization.
Although this experimental method is used for characterizing acoustic materials, in this work the main interest is in the behaviour of the waves inside the tube. By means of a harmonic analysis, the resonant frequencies of the tube are determined. Software ANSYS, based on the finite element method, has been used for this purpose. The influence of varying the mesh density in the model has been analyzed.
The standing wave tube follows the requirements of the Standard ISO 10534-2 (Determination of sound absorption coefficient and impedance in impedance tubes. Part 2: Transfer-function method) [1]. It is a rigid-walled tube of constant circular cross section made of methacrylate.
The obtained results in the numerical simulation are compared with those obtained experimentally with the transfer function method. The experimental measurements have been studied with different distances between microphones. The influence of this distance has been evaluated.

2. RESONANT FREQUENCIES IN DUCTS

2.1. DUCT CLOSED AT ONE END

Figure 1 shows the scheme of the standing wave tube that is studied. The rigid-walled tube is excited at one end and is closed at the other one.

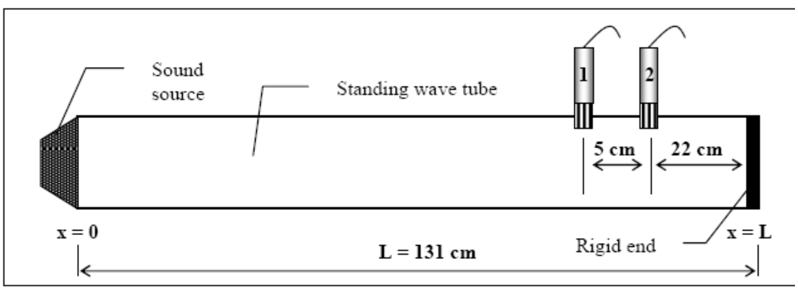

Figure 1. Standing wave tube.

A piston vibrating harmonically at low frequency is considered, so that only plane waves can propagate inside the tube. The tube is excited at x = 0 and is rigidly terminated at x = L. The equation 1 shows the resonant condition [2].

$$\frac{Z_{m0}}{\rho_0 \cdot c \cdot S} = -j \cdot \cot(k \cdot L) \quad (1)$$

Z_{m0}, is the mechanical impedance at x = 0; ρ_0 is the air density; *c,* is the sound speed in the air; *S,* is the cross sectional area of the tube; *k,* is the wavenumber; *L,* is the length of the tube.
The reactance is zero when *cot (k·L)* = 0. The resonant frequencies are calculated with equation 2:

$$f_n = \frac{(2 \cdot n - 1)}{4} \cdot \frac{c}{L} \quad (2)$$

Therefore, the resonant frequencies are the odd harmonics of the fundamental one. The tube excited at one end and rigidly terminated at the other one has a pressure node at x = 0 and a pressure antinode at x = L. Table I shows the resonant frequencies in the range from 0 to 1000 Hz.

Table I.- Resonant frequencies of the tube

Nº	**(2·n-1)**	**f_n (Hz)**
1	1	**65'49**
2	3	**196'48**
3	5	**327'48**
4	7	**458'47**
5	9	**589'46**
6	11	**720'46**
7	13	**851'45**
8	15	**982'44**

Figure 2 shows the device used for measuring the resonant frequencies in a standing wave tube.

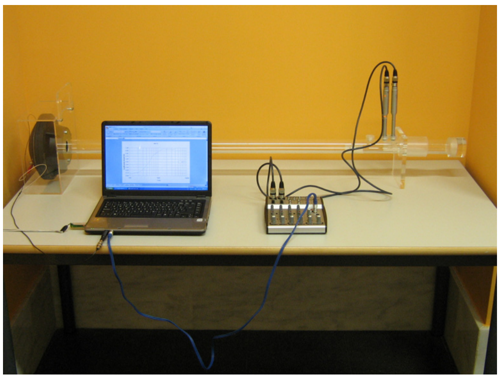

Figure 2. Experimental measurements

Several measurements with different distances between microphones have been done with the purpose of studying the influence of this distance in the obtained results. Table II shows the obtained results with a MATLAB function based on the transfer function method [3].

Table II.- Frequencies with different distances between microphones

Frequency with theoretical equation	Frequency (Hz) Distance between micros: 3’5 cm	Frequency (Hz) Distance between micros: 5 cm	Frequency (Hz) Distance between micros: 8’5 cm
65’49	65’50	65’40	65’50
196’48	196’40	196’40	196’50
327’48	327’25	327’25	327’35
458’47	458’25	458’23	458’30
589’46	589’08	589’00	589’25
720’46	720’06	720’06	720’16
851’45	850’93	850’93	851’03
982’44	981’88	981’85	981’98

2.2. DUCT WITH A SOUND ABSORBENT MATERIAL AT ONE END

Now, the resonant frequencies in a standing wave tube with a sample of rock wool in one end have been evaluated experimentally. It is the same tube used previously. Figure 3 shows a graph with the resonant frequencies obtained experimentally.

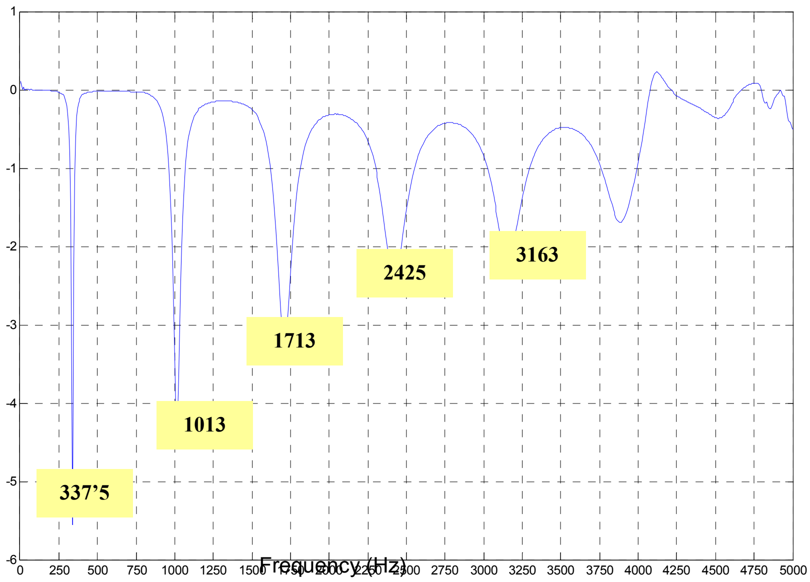

Figure 3. Resonant frequencies in a standing wave tube with a sample of rock wool at one end. This graph relates the transfer function (H_{12}) with the frequency.

3. NUMERICAL SIMULATION

ANSYS software based on the finite element method has been used to develop the numerical simulation. In ANSYS there are two specific acoustic elements: FLUID29 (for 2D models) and FLUID30 (for 3D models). These elements are used to model the fluid portion and they accept fluid density and speed of sound as input data [4]. In this work an axys-simmetric 2D model has been developed. The use of a 2D model reduces analysis time compared to an equivalent 3D model [5].

FLUID29 element is defined by four nodes with three degrees of freedom at each node: translations in “x” and “y” axis and a reference pressure. The reference pressure is used to calculate the sound pressure level. The acoustic pressure in the fluid is determined by the wave equation:

$$\frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 0 \tag{3}$$

P is the sound pressure and *t* is the time.
There are some assumptions: the fluid is compressible, the fluid is inviscid (no viscous dissipation) and the density is uniform throughout the fluid.

3.1. DUCT CLOSED AT ONE END

The model has been evaluated with three different mesh densities: 262, 524 and 1048 elements. A harmonic analysis has been conducted in a frequency range from 0 to 1000 Hz. Table III shows the characteristics of the finite element model.

Table III.- Characteristics of the ANSYS model

Length	**131 cm**
Diameter	**4 cm**
Air density	**1’18 kg/m^3**
Sound speed	**343’2 m/s**
Reference pressure in the air	**20·10^{-6} Pa**
Pressure at the sound source	**1 Pa**
Frequency range	**0 – 1000 Hz**

Figure 4 shows the results of the harmonic analysis at microphone 2 position (at 22 cm from the rigid termination, Figure 1).

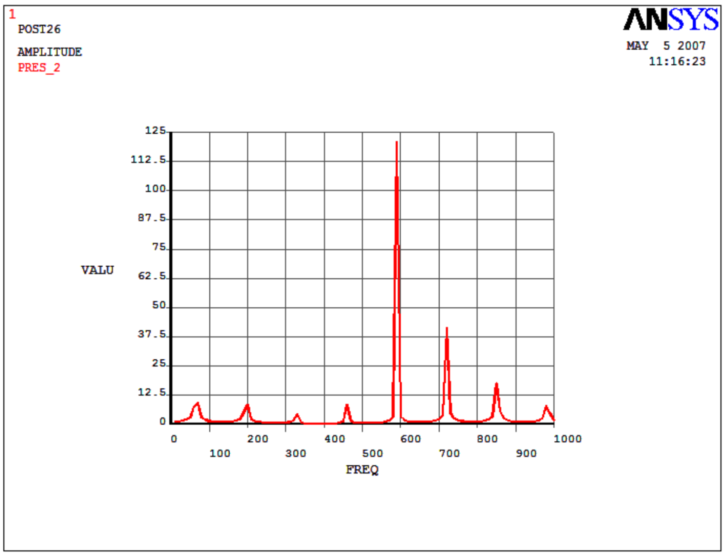

Figure 4. Harmonic analysis at microphone 2 position

Table IV shows the resonant frequencies obtained with the theoretical equation and those obtained varying the mesh density in the numerical model.

Table IV.- Frequencies with different mesh densities

Theoretical equation	Numerical simulation 262 elements	Numerical simulation 524 elements	Numerical simulation 1048 elements
65’49	65’49	65’49	65’49
196’48	196’53	196’50	196’49
327’48	327’67	327’53	327’49
458’47	459’00	458’61	458’51
589’46	590’59	589’75	589’54
720’46	722’52	720’98	720’59
851’45	854’85	852’31	851’67
982’44	987’67	983’77	982’77

3.2.- DUCT WITH A SOUND ABSORBENT MATERIAL AT ONE END

In order to simulate dissipative acoustic filters with a sound absorbent material inside the expansion chamber, a first step is developed in this work. So, a model in which the porous material is considered as a fluid, has been developed. This porous material is a sample of rock wool.

The sound speed through the rock wool is determined using the mathematical model of Delany and Bazley [6]. Equation 4 is used to calculate the sound speed in the rock wool.

$$\frac{\omega}{v_m} = \frac{2 \cdot \pi \cdot f}{c_0}\left(1 + 0'0978 \cdot \left(\frac{\rho_0 \cdot f}{\sigma}\right)^{-0'7}\right) \quad (4)$$

v_m, is the sound speed through the material; c_0, is the sound speed in the air; f, is the frequency; ρ_0, is the air density; σ, is the specific flow resistance.

From equation 4, it is obtained this equation:

$$v_m = \frac{c_0}{\left(1 + 0'0978 \cdot \left(\frac{\rho_0 \cdot f}{\sigma}\right)^{-0'7}\right)} \quad (5)$$

Taking f = 2500 Hz (average value in the studied frequency range) and $\sigma \approx$ 60400 Rayls/m [7]:

$$v_m = 190 m/s$$

Table V shows the characteristics of the rock wool.

Table V. Characteristics of rock wool

ROCK WOOL	
Thickness	1,5 cm
Density	150 kg/m^3
Sound speed in the rock wool (at 2500 Hz)	190 m/s

Now, the resonant frequencies in the standing wave tube with a sample of rock wool at one end, are evaluated by means of the numerical simulation (Figure 5).

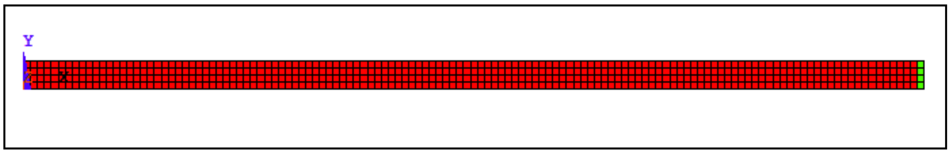

***Figure 5. Numerical model of a duct with a sample of rock wool at one end*.**

Table VI shows the characteristics of this model.

Table VI. Input data of the numerical model (2D)

Length	**1,31 m**
Diameter	**0,04 m**
Air density	**1,18 kg/m^3**
Sound speed in air	**343,2 m/s**
Reference pressure in air	**20·10^{-6} Pa**
Density of the rock wool	**150 kg/m^3**
Sound speed through the rock wool	**190 m/s**
Thickness of the sample of rock wool	**1,5 cm**
Frequency range	**0 – 5000 Hz**
Pressure at the sound source	**1 Pa**

Figure 6 shows the resonant frequencies obtained with the numerical modal analysis.

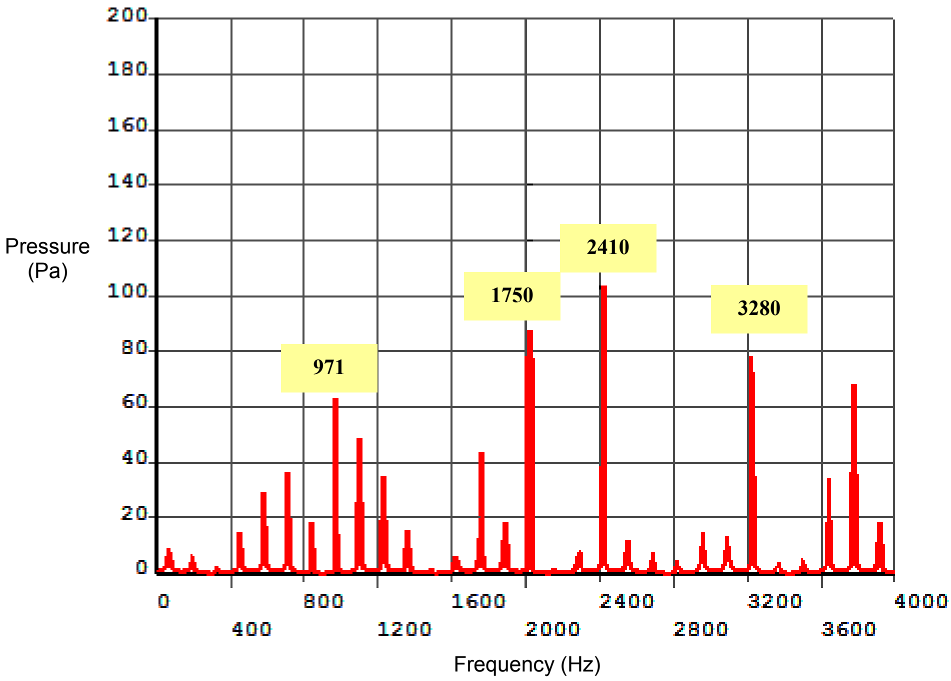

Figure 6. Resonant frequencies obtained with the numerical simulation.

This graph shows that the experimental resonant frequencies and the numerical ones are a little different.

4. CONCLUSIONS

This work is considered as the starting point of a more detailed study in which it is intended to define a numerical model for simulating the tests that Standard ISO 10534-2 proposes in order to characterize different porous and fibrous materials used in the scope of the architectural acoustics.

The next step will be to develop a numerical model to evaluate dissipative acoustic filters. The dissipative acoustic filters work well at high frequencies and there is a sound absorbent material inside the expansion chamber.

On the other hand, having into account that the sound speed in the material depends on the frequency (in the numerical simulation an average value has been considered), these results can be considered sufficiently accurate to validate this numerical model.

There are many factors that have an influence in the accuracy of the finite element model; in this study, however, a first approach of the influence of the mesh density in the obtained results has been analyzed and it has been compared with the experimental results.

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