FINITE ELEMENT APROACH TO A COUPLED STRUCTURAL ACOUSTIC SYSTEM: APPLICATION TO A LOUDSPEAKER CABINET

SEGURA ALCARAZ[1], J.; GADEA BORRELL[1], JULIÁ SANCHIS, E.; J. M.; MASIÁ VAÑÓ[1], J.; RAMIS SORIANO[2], J.[1]
Escuela Politécnica Superior de Alcoy - Plaza de Ferrándiz y Carbonell, s/n. Alcoy (03801)
jsegura@mcm.upv.es; jmgadea@mes.upv.es
Escuela Politécnica Superior. Universidad de Alicante. Campus de San Vicente del Raspeig, (03690). jramis@ua.es

Keywords: loudspeaker enclosure, modal analysis, finite elements method.
Abstract The object of the present work is to make a contribution to the quantification of the vibration behavior of the walls of two loudspeaker prototypes mounted in closed boxes, made of different materials. Using the finite elements method, a coupled fluid-structure numerical model of a loudspeaker-box system was implemented. The numerical model was calibrated from the experimental results obtained on a real model by means of modal analysis, vibration and pressure measurements. The conclusion is that the sonorous response of the system is affected by the vibration behavior of the box and its material.

1 INTRODUCTION

In a sound system, the loudspeaker or transductor transforms the electrical waves into mechanical energy, and then the mechanical energy into acoustic energy via the diaphragm. When the diaphragm of a loudspeaker begins to move, it works like a piston at low frequencies, creating a pressure field in the air inside and outside the speaker cabinet. At certain frequencies, the interior air can begin to resonate, creating a series of stationary waves that follow a specific vibration patterns [5].
The walls of the cabinet are not infinitely rigid and can vibrate for two basic reasons: the pressure generated inside can cause a forced vibration or excite resonance of the cabinet walls. Secondly, a loudspeaker that is mounted on one of its walls directly transmits vibration to the cabinet, also causing forced vibration of the walls or exciting resonance within them [7] [8]. The principal elements of the system; interior air, cabinet and exterior air constitute a totally coupled unit whose behavior determines the sound transmission to the exterior.
The problems associated with designing sound broadcasting systems, such as a closed box loudspeaker, are often tackled using circuits with electromechanics-acoustic analogues [9], [10]. However, these types of models cannot consider the collective effect of the high order modes of the basic elements of the system: loudspeaker, cabinet, interior and exterior air [12], [3]. In this work, we present a numerical model which considers the real geometry of the system and its principal dynamic characteristics, such as rigidity, mass and damping capacity, in a spatial way. The numerical model was used to parametrically study the influence of the material used in the cabinet on the vibration and acoustic response of the system as a whole.
The Finite Element method has been used successfully in various research works, [2], [11], [6], [1], to study the speaker cone or the resonance in the air inside the cabinet. Other research works, [3], [4], have highlighted the suitability of the Finite Element Method when compared with other methods, such as the Boundary Element Method, or the Finite Differences Method for studying problems associated with these types of fluid-structure coupled systems in acoustics at low frequencies.

2 DEVELOPMENT

For the enclosed cabinet models, we used two boxes of identical dimensions, one made of medium density fiber (MDF) sheets, and one of Polymethylmethacrylate (PMMA). Figure 1 shows the dimensions of the models used in the experiments, as well as an image of the MDF cabinet.

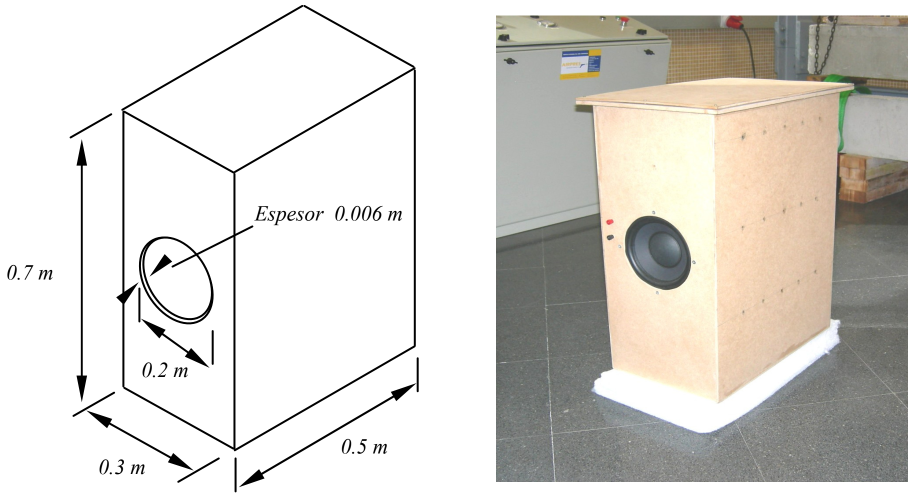


Figura 1 Dimensions of the cabinet models.

The choice of two different materials with the same dimensions of internal cavity allowed a comparison of the results obtained from each experiment, highlighting the influence of each material on the vibro-acoustic behavior of the system, independently of the internal air resonance. The materials were characterized experimentally. Table 1 shows the results obtained.

DM			*PMMA*		
Density (kg/m^3)	*E (MPa)*	*Loss Factor*	*Density (kg/m^3)*	*E (MPa)*	*Loss Factor*
870	*2699.74*	*0.082*	*1150*	*3611.56*	*0.117*

Tabla 1. Properties of the materials used for the boxes.

Free boundary conditions were used for the models because these conditions are easy to reproduce in the finite element numerical models, and the same conditions were used for all measurements and experimental procedures. We chose to support the models with a low Young module support, in concrete with a foam textile material.

2.1 EXPERIMENTAL PROCEDURE

In order to achieve the objectives proposed, we took a series of experimental measurements from closed box loudspeaker models. In the experimental phase we studied the vibration behavior of the walls of the structure using modal analysis and vibration measurements, and the sound response of the coupled system using measurements of pressure and acoustic intensity. Figure 2 shows schemes of the experimental set-ups used. The numbering corresponds to the following elements: 1.- cabinet model, 2.- data acquisition card, 3.- accelerometer, 4.- impact hammer , 5.- PC, 6.- sound Intensity Probe, 7.- amplifier and 8.- anechoic chamber.

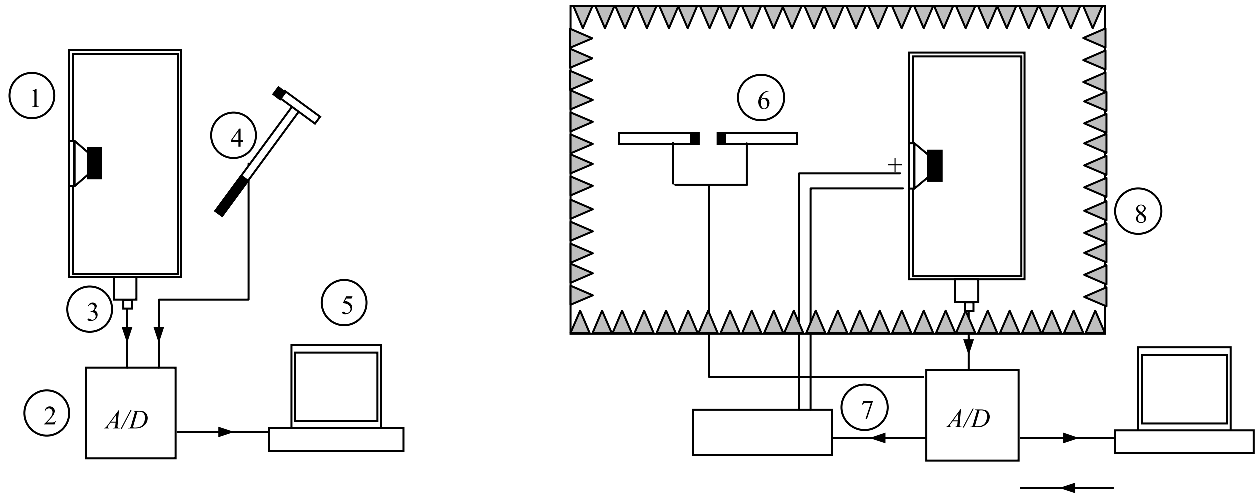


Figura 2 Experimental set-up.

The vibration measurements and the modal analysis were restricted to the 0-200 Hz range, because in this range of frequencies the first vibrations in the box are found. The results of the modal analysis reflect a great number of vibration modes for both models in this range of frequencies. Both with the MDF model and the PMMA model, the vibration measurements showed areas of wider ranges of vibration velocities. These areas were: 50-55, 90-115 and 130-140 Hz for the rear and side walls, and 80-95, 110-115 and 120-130 Hz for the front wall. These frequency bands coincide with the relative modal density of the box. Table 2 shows the results from the modal analysis and the velocity measurements in the box. Also shown in the table are the vibration frequencies of the air inside the cavity obtained with the equation:

$$\omega_{n_x,n_y,n_z} = \frac{c_0}{2}\sqrt{\left(\frac{n_x}{l_x}\right)^2 + \left(\frac{n_y}{l_y}\right)^2 + \left(\frac{n_z}{l_z}\right)^2} \qquad (1)$$

where ω_{n_x,n_y,n_z} *(Hz)* are the frequencies of the vibration modes of the interior air, l_x, l_y, l_z *(m)* are the dimensions of the cabinet in the directions *x, y, z* respectively, and n_x, n_y, n_z can take the values 0,1,2…etc. and define the number of the mode.

	MDF Enclosure		PMMA Enclosure		Enclosure inside air	
Enclosure side	High velocity frequency Band (Hz)	Frequency modes(Hz)	High velocity frequency Band (Hz)	Frequency modes(Hz)	Mode (nx, ny, nz)	Frequency(Hz)
Lateral	50-55	52.9	40-70	42.2, 56.4	1,0,0	245.71
	90-120	84.0, 96.4, 119.2	90-115	93.3, 103.5, 107.8	0,1,0	344.00
	125-145	123.4, 150.7	130-140	124.2, 143.8	1,1,0	422.74
Rear	50-70	53.6, 74.6	50-55	42.3, 59.2	2,0,0	492.42
	90-115	89.4, 122.2	90-115	87.2, 98.4	0,0,1	573.33
	130-145	133, 145	130-140	125, 145	2,1,0	599,86
Front	40-80	45.4, 68.3, 80.7	45-80	48.3, 81.3	1,0,1	623.76
	120-130	115.1, 128.4	100-140	98.4, 117.3, 137.1	0,1,1	668.61
	180-200	175.8	-	-	0,2,0	688.00
	-	-	-	-	1,1,1	712.33
	-	-	-	-	1,2,0	730.56
	-	-	-	-	0,2,0	737.14
	-	-	-	-	2,0,1	755.12
	-	-	-	-	3,0,1	813.45
	-	-	-	-	2,1,1	829.78
	-	-	-	-	2,2,0	845.48
	-	-	-	-	0,2,1	895.57
	-	-	-	-	1,2,1	928.67
	-	-	-	-	3,0,1	933.85

Table 2 Experimental results from the modal analysis and vibration velocity.

Figure 3 shows the results of the sound intensity measurements where we can see that the contribution of the front walls is similar for both materials, while the contribution of the side and rear walls to the sound transmission is greater with the MDF than with the PMMA.

To summarize, the experimental results show that the first modes of the boxes are to be found in the frequency interval 0-200 Hz. From 200 to 1000 Hz the first interior air modes appear, and from 1000 Hz the modal density increases and the loudspeaker ceases to function like a piston. We can conclude that from 0 to 200 Hz, the signal becomes “colored”, principally due to the vibration modes of the enclosure, given that the first vibration mode is to be found at 245 Hz.

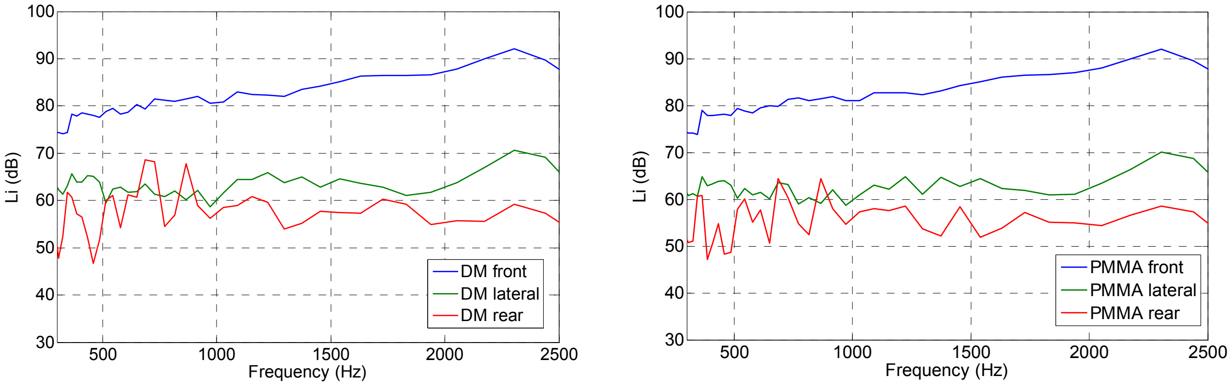
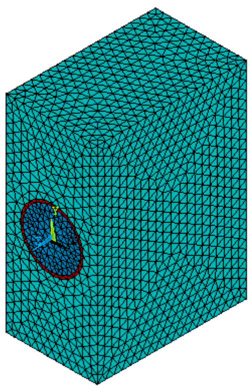


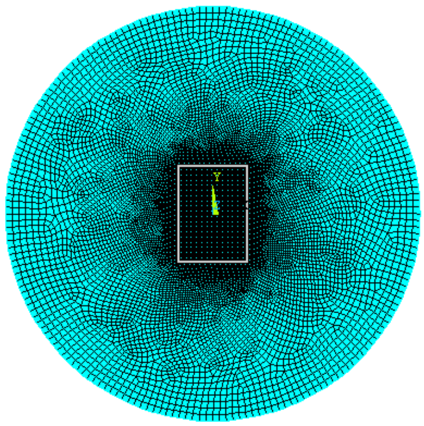
Figure 3 Results of the sound intensity measurements.

2.2 FINITE ELEMENT NUMERICAL MODEL

Based on experimental results, two finite element numerical models were used. One was a three-dimensional model with fluid-structure coupling between the box, the piston and the air inside the box. This model was used to focus on the vibration behavior of the box at low frequency, from 0 to 200 Hz. The other was a two-dimensional model with fluid structure coupling between the box, the piston and both interior air and exterior air. This model was designed to discover the effect that the box and resonance in the interior air has on the sound pressure transmitted by the system in the band of frequencies between 0 and 1000 Hz. Figure 4 shows the finite element numerical model matrix.

Figure 4a shows the three-dimensional model with the following elements: the enclosure, diaphragm or piston, elastic band around the piston acting as a compliance and the internal air of the box. Figure 4b shows the two-dimensional model which, as well as the elements shown in 4a, also shows the air exterior to the box limited by an infinite acoustic absorption boundary.





4a ***4b***

Figure 4 3D and 2D models used.

To simulate the movement of the cone, a harmonic force was applied to the center of the membrane that constitutes the diaphragm of the loudspeaker. Around this membrane, we placed in both models a ring of elastic material in order to study the effect of the compliance of the loudspeaker. In the absence of fluid acoustics, the equation for the harmonic problem is the following:

$$[M]\{\ddot{u}^*\}+[C]\{\dot{u}^*\}+[K]\{u^*\}=\{f^{ext}(t)\} \tag{2}$$

where $[M]$ is the matrix of the mass of the structure, $[C]$ is the damping matrix of the structure, $[K]$ is the matrix of the structure's rigidity, $\{f^{ext}(t)\}$ are the exterior forces applied to the structure and $\{u^*\}$ is the nodal displacement vector which constitutes the unknowns in the equation.

The displacements in the structure cause the pressures in the fluid and vice versa. The equations which govern the coupling between the fluid and the structure possess the two degrees of freedom, displacements and pressures, which will be common in the interphase of both. If we add the forces due the pressure of the fluid in the interphase to the movement equation of the structure, we obtain:

$$[M]\{\ddot{u}^*\}+[C]\{\dot{u}^*\}+[K]\{u^*\}=\{f^{ext}(t)\}+\{f^{pres}(t)\} \tag{3}$$

where $\{f^{pres}(t)\}$ is a vector which represents the pressure load due to the fluid of the interphase.

In the equation for the movement of the fluid, the coupling matrix $[R]$ appears, whose function is the coupling in the interphase between fluid and structure.

$$[M^F]\{\ddot{p}^*\}+[K^F]\{p^*\}+\rho[R]\{\ddot{u}^*\}=0 \tag{4}$$

where $[M^F]$ is the fluid mass matrix, $[K^F]$ is the fluid rigidity matrix, $\{u^*\}$ and $\{p^*\}$ are the nodal pressure and displacement vectors, which constitute the unknowns of the equation.

The matrix $[R]$ relates the pressure load of the fluid in the structure with the fluid pressures:

$$\{f^{pres}(t)\}=[R]\{p\} \tag{5}$$

Equations (3) y (4) are written in just one equation, and taking into account relation (5), we obtain the following equation which governs the behavior of the fluid-structure coupled system [13]:

$$\begin{bmatrix}[M] & [0]\\ [\rho[R^T]] & [M^F]\end{bmatrix}\begin{Bmatrix}\{\ddot{u}^*\}\\ \{\ddot{p}^*\}\end{Bmatrix}+\begin{bmatrix}[C] & [0]\\ [0] & [0]\end{bmatrix}\begin{Bmatrix}\{\dot{u}^*\}\\ \{\dot{p}^*\}\end{Bmatrix}+\begin{bmatrix}[K] & [-R]\\ [0] & [K^F]\end{bmatrix}\begin{Bmatrix}\{u^*\}\\ \{p^*\}\end{Bmatrix}=\begin{Bmatrix}\{f^{ext}(t)\}\\ \{0\}\end{Bmatrix} \tag{6}$$

The numerical model was calibrated from the experimental data. Figure 5 shows the comparison between the level of real and simulated sound pressure from the numerical model for both cabinet materials, MDF and PMMA.

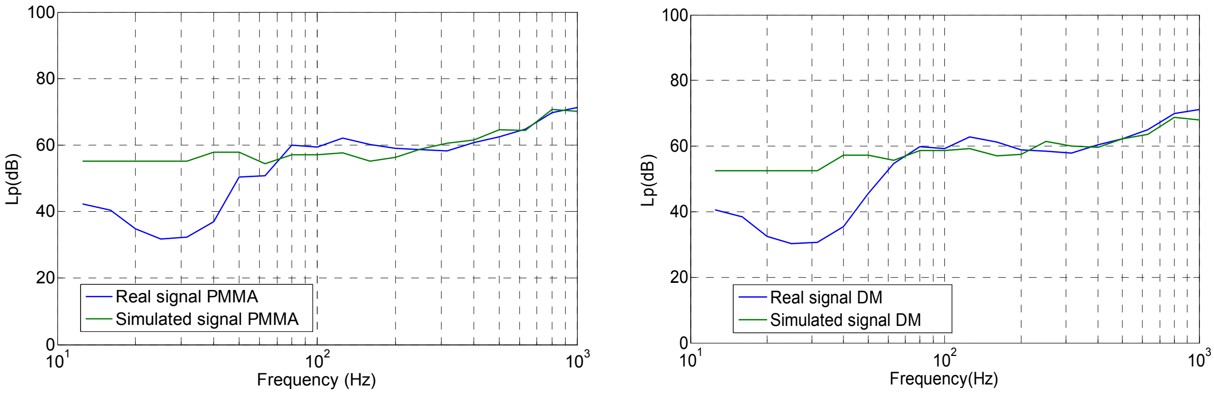


Figure 5. Comparison of real and simulated sound pressure level.

Figure 6 shows the comparison between the real signal and the simulation of the average velocity of the lateral walls for the two types of cabinet, MDF and PMMA.

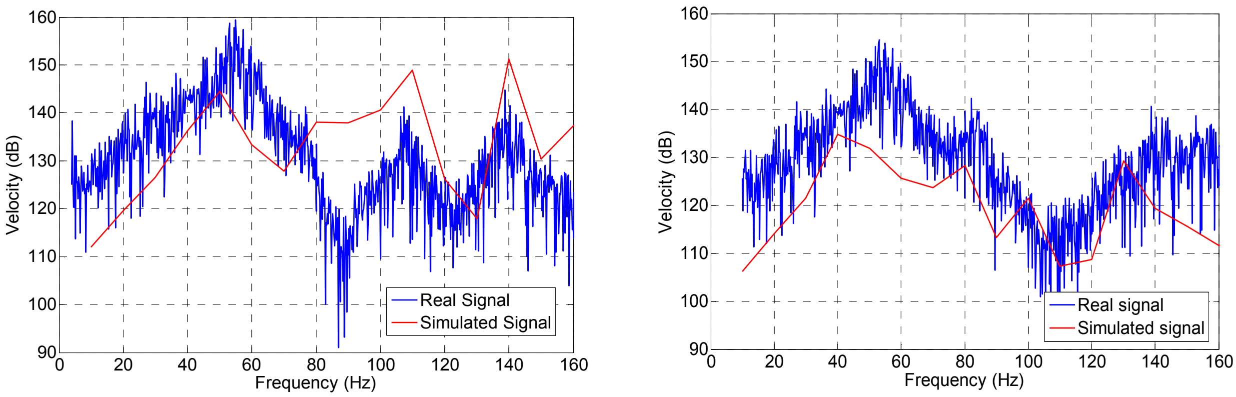
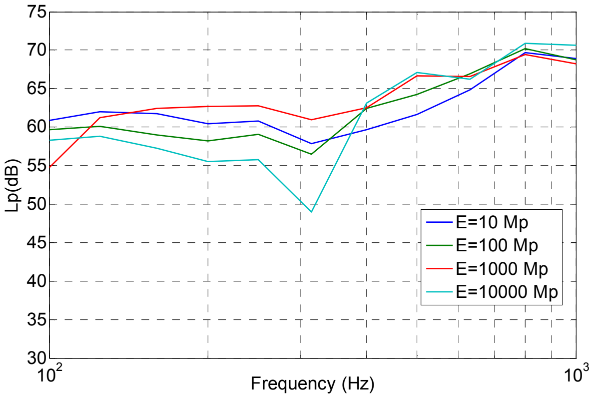
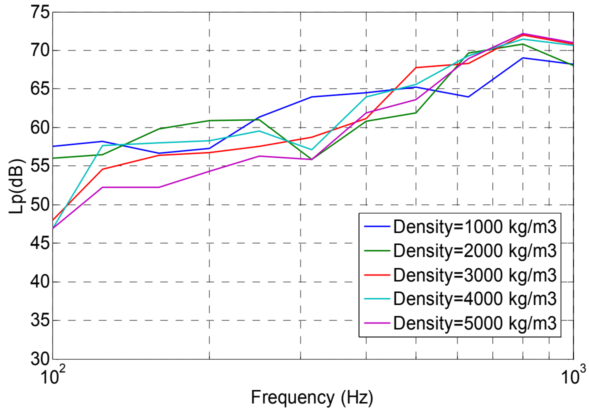


Figura 6. Comparison of real and simulated of the average velocity.

Once the model was calibrated, we carried out a parametric analysis with the aim of studying the influence of the cabinet materials characteristics on the vibro-acoustics of the system. Figure 7a shows the simulation results of the effect that the Young's module of the material has on the sound pressure transmitted by the system. Figure 7b shows the simulation results of the effect that the variation in density of the enclosure material has on the sound pressure transmitted by the system.



7a



7b

Figure 7. Parametric study: influence of Young module and material density

3 CONCLUSIONS

In general, the vibro-acoustic behavior of the system depends, in great part, on the dimensions and geometry of the structure. These factors condition the dynamic behavior of the walls of the enclosure and of the interior air. It is reccommendable to carry out a pre-design in order to separate, in the same range of frequencies, the structural resonances of the walls from the resonances of the interior cavity air, to avoid coupling between the two can take place.

It is the first vibration modes of the cabinet that most influence the sonorous response of the system at low frequencies. As we increase the mode number with the frequency, the modal shapes shows a minor vibrating surface in phase, meaning that its contribution to the sound field of the system also diminishes.

The results of the parametric study carried out using the numerical models show the importance of the material of the cabinet in the sonorous response of the system:

- The variations in the density of the material from which the cabinet is made influences the sonorous response of the coupled system at low frequencies, where resonance in the walls of the structure are more important. From 500-600 Hz onwards, the signal stabilizes and the density of the material loses its influence on the sound pressure level of the system.
- The variations in the Young model of the cabinet material also have a marked influence on the sonorous response of the system at low frequencies, due to resonance in the structure. The same as occurred with density, from 500Hz onwards, the contribution of resonance in the cabinet loses relevance in the face of the forced vibrations due to stationary waves.

  The study of vibrations in the walls of the cabinet with the variation of the Young moduli indicates that on increasing this module, the resonance frequencies of the structure also increase. Thus, we can conclude that the variation in rigidity of the structure, as the rigidity of the material increases or when supporting bars are used, does not eliminate resonance in the cabinet but rather displaces them in frequency. These results agree with the results of research carried out by Bastar, K.J. y Capone, D.E. [1].

We can conclude that the numerical models presented, with their entry simplifications and limitations, constitute a flexible design tool, which allows the fast and economical study of the importance of the cabinet structure and the intrinsic properties of its construction materials, in the sonorous response of the system.

REFERENCES

[1] Bastyr, K.J. y Capone, D.E. ON THE ACOUSTIC RADIATION FROM A LOUDSPEAKER´S CABINET. *Journal of the Audio Engineering Society*, Vol 51(4), 2003, pp. 234-241.

[2] Kagawa, Y. et alt. FINITE ELEMENT APROACH TO A COUPLED STRUCTURAL ACOUSTIC RADIATION SYSTEM WITH APPLICATION TO A LOUDSPEAKER. *Journal of sound and Vibration*, Vol 69, 1980, pp.229-243.

[3] Karjalainen, M. et alt. COMPARISON OF NUMERICAL SIMULATION MODELS AND MEASURED LOW-FREQUENCY BEHAVIOUR OF LOUDSPEAKER ENCLOSURES. *Journal of the Audio Engineering Society*, Vol 49(12), 2001, pp. 1148-1166.

[4] Kirkuk, S. M y Jones, M.A. COMPUTATIONAL METHODS FOR THE ACOUSTIC MODAL ANALYSIS OF AN ENCLOSED FLUID WITH APPLICATION TO A LOUDSPEAKER CABINET. *Applied Acoustics*, Vol 48(4), 1996, pp. 275-299.

[5] Runnemalm, A. STANDING WAVES IN A RECTANGULAR SOUND BOX RECORDED BY TV HOLOGRPHY. *Journal of Sound and Vibration,* Vol 244(4), 1999, pp. 689-707.

[6] Sakai, S. et alt. ACOUSTIC FIELD IN AN ENCLOSURE AND ITS EFFECTS ON SOUND-PRESSURE RESPONSES OF A LOUDSPEAKER. *Journal of the Audio Engineering Society*, Vol 32(4), 1984, pp. 218-227.

[7] Skrodzka, E.B. AN INFLUENCE OF AN ENCLOSURE OF MODAL BEHAVIOUR OF LOUDSPEAKERS. *Journal of the Acoustic Society of Japan,* Vol 20(4), 1999, pp. 261-270.

[8] Skrodozka, E.B. y Sek, A.P. COMPARISON OF MODAL PARAMETERS OF LOUDSPEAKERS IN DIFFERENT WORKING CONDITIONS. *Applied Acoustics*, Vol 60(3), 2000, pp. 267-277.

[9] Small, R. H. CLOSED BOX LOUDSPEAKER SYSTEMS: PART I: ANALISIS. *Journal of the Audio Engineering Society*, vol. 20(10), 1972, pp. 798-808.

[10] Small, R. H. CLOSED BOX LOUDSPEAKER SYSTEMS: PART II: SYNTHESIS. *Journal of the Audio Engineering Society*, vol.21(1), 1973, pp. 11-18.

[11] Suzuki, K.; Nomoto, I. COMPUTERIZED ANALYSIS AND OBSERVATION OF THE VIBRATION MODES O A LOUDSPEAKER. *Journal of the Audio Engineering Society*, Vol 30, 1982, pp. 96-106.

[12] Tappan, P.W. LOUDSPEAKER ENCLOSURE WALLS. *Journal of the Audio Engineering Society*, Vol 10(3), 1962, pp. 224-231.

[13] Zienkiewicz, O.C. y Bettess, P. FLUID–STRUCTURE DYNAMIC INTERACTION AND WAVE FORCES. INTRODUCTION TO NUMERICAL TREATMENT, *Int. J. Numer. Methods Eng*., vol (13), 1978, pp. 1–16.