

## **THE DESIGN OF A DIGITAL VIBRATIONS SUPPRESSION CONTROLLER FOR MULTIMODE MECHATRONIC SYSTEMS**

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**Keywords:** vibrations suppression, digital compensator, mechatronic systems

**Abstract:** This paper presents the design of a multimode vibrations suppression compensator for numerical controlled mechatronic systems. The compensation strategy requires knowledge about the modal behavior of the non-compensated system and a reference model. The non-compensated system's modes are defined by second order discrete transfer functions. The reference model has the same modal structure and parameters as the non-compensated system, except for the damping ratios that are 1. The compensator is the result of the division of the reference model by the non-compensated model. Two examples of such compensators are shown, one for singlemode and one for multimode systems. The method in this paper is focused on discrete time systems suitable for implementation on DSPs and numerical computers.

### **1. INTRODUCTION**

The mechanical vibrations have intensively been studied both for understanding their behavior [5] and to suppress their undesired effects over the mechanical structures [1] [6]. There are of course applications in which the controlled vibrations are intentionally generated, but these are not covered in this paper.

Mechatronics is the science of intelligent mechanical systems, but mechatronics systems are still subject to mechanical vibrations. The vibrations' effects can severely alter the performance of mechatronic systems, such as positioning precision and processing speed. The purely mechanical vibrations suppression methods usually require altering the original structure of the system by adding various elements such as vibrations absorbers, dampers or oscillating masses [6]. These methods can be efficient in suppressing the residual vibrations but they can also change the original performances of the system. They have very limited flexibility, lifetime and usually high implementation and maintenance costs.

With the advent of the electromechanical and mechatronic systems some variations of the previous solutions have been developed, such as the active force devices. Their performances are highly improved. However, due to their intrinsic mechanical components are also subject to fatigue and maintenance costs.

A more flexible category of non-invasive techniques has become available and benefits of the computing features of the mechatronic systems. The most prominent representatives of this category are the command preshaping techniques such as Posicast [4], input shaping [8] and general purpose filter based preshaping [3]. These methods avoid the problems of the mechanical solutions.

The time delay based methods such as input shaping can achieve very good results as show in [9]. However, they require a priori knowledge of the system's structure and behavior, thus being sensitive to parametric errors. Various improvements of the original methods have been introduced to achieve a much better robustness [10].

This paper presents a command preshaping technique based on a reference model and the identification of the modal behavior of the non-compensated system. The resulting compensator is designed in discrete time, thus being suitable for implementation in digital systems. For the interested reader, a continuous version of the compensator can be designed as shown in [7]. Some detailed applications for a single mode and a multimodal system are also provided.

## 2. DISCRETE TIME MULTIMODE NON-COMPENSATED SYSTEM MODEL

The original non-compensated mechatronic system is mechanical by excellence and therefore is a subject to a certain amount of elasticity. This inevitably leads to some undesired structural vibrations.

In order to study the modal behavior of the system, this method requires using a second order differential equation for each mode. This means that an independent mode can be written in a more general manner as shown in Eq. (1).

$$\frac{d^2 y}{dt^2} + 2 \cdot \xi_i \cdot \omega_{ni} \cdot \frac{dy}{dt} + \omega_{ni}^2 \cdot y = K_i \cdot \omega_{ni}^2 \cdot u \quad (1)$$

where,  $K_i$  – the gain of the  $i^{\text{th}}$  mode;  $\omega_{ni}$  – the natural undamped frequency of the  $i^{\text{th}}$  mode (in rad/sec);  $\xi_i$  – the damping ratio of the  $i^{\text{th}}$  mode;  $y$  – the output signal (position);  $u$  – the input signal. Solving this equation may be relatively easy for a second order equation but for higher orders this can prove difficult and inefficient.

A simpler approach is to convert the equation using the Laplace transform, thus obtaining the continuous time equation that can be rewritten in the transfer function form as in Eq. (2).

$$H_i(s) = \frac{K_i \cdot \omega_{ni}^2}{s^2 + 2 \cdot \xi_i \cdot \omega_{ni} \cdot s + \omega_{ni}^2} \quad (2)$$

This is acceptable for the simulation purposes, but the modern numerical computation systems require a discrete time form of this transfer function that can be easily obtained from Eq. (1), with several simplifications in mind. The resulting discrete time transfer function for one mode is given in Eq. (3), where  $t_s$  is the sampling period. The sampling period may play a crucial role in the discrete model, but this discussion is not part of the present paper.

$$H_i(q^{-1}) = \frac{K_i \cdot \omega_{ni}^2 \cdot t_s^2}{(1 + 2 \cdot \xi_i \cdot \omega_{ni} \cdot t_s + \omega_{ni}^2 \cdot t_s^2) - 2 \cdot (1 + \xi_i \cdot \omega_{ni} \cdot t_s) \cdot q^{-1} + q^{-2}} \quad (3)$$

Knowing the independent mode behavior, the multimodal system can be represented as a parallel connection of these modes. Using the well known method of composition in the systems' theory, the resulting transfer function can be written as:

$$H(q^{-1}) = \sum_{i=1}^n H_i(q^{-1}) \quad (4)$$

In a more detailed form, Eq. (4) can be rewritten as Eq. (5). This variant is suitable for digital computers implementation.

$$H(q^{-1}) = \frac{\sum_{i=1}^n \left( num\{H_i(q^{-1})\} \cdot \prod_{j=1, j \neq i}^n den\{H_j(q^{-1})\} \right)}{\prod_{i=1}^n den\{H_i(q^{-1})\}} \quad (5)$$

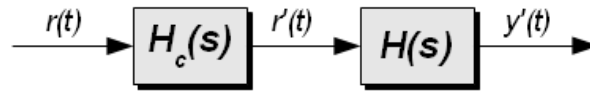
where,  $num\{H(q^{-1})\}$  – the transfer function's numerator;  $den\{H(q^{-1})\}$  – the transfer function's denominator.

Eq. (5) depicts the discrete time model of the original mechatronic system embedding the multimodal behavior that must be controlled.

## 3. VIBRATIONS SUPPRESSION COMPENSATOR DESIGN

The compensator design method in this paper is based on enforcing the compensator transfer function to reshape the original reference signal in order to trigger a vibrations free output, as shown in Fig. 1. This requires an accurate model of the original system, as described in the previous section, and a reference model that is tailored to meet the goal

of no residual vibrations in the system's output.



**Fig. 1 – The compensation structure**

It is important to notice that this approach does not address the outside noise but the intrinsic vibrations caused by the structure of the system. Therefore, in order to have an automatic controlled system, a closed-loop must be used. However, the compensator design does not care about the status of the loop (opened or closed), the method being the same.

The most important part of the compensator design is choosing the appropriate reference model transfer function. This is intended to be the actual behavior of the whole system. There are many ways to choose this function. However, they all have benefits and disadvantages.

This paper proposes using a reference model with the exact same structure as the original model and even more, with the same parameters' values, except for the damping ratios that must be 1. Therefore, the reference model is build as a parallel connection of single mode transfer functions as in Eq. (6).

$$H_{Ri}(q^{-1}) = \frac{K_i \cdot \omega_{ni}^2 \cdot t_s^2}{[(1 + \omega_{ni} \cdot t_s) - q^{-1}]^2} \quad (6)$$

The complete transfer function can be obtained by the formula given in Eq. (7).

$$H_R(q^{-1}) = \frac{\sum_{i=1}^n \left( num\{H_{Ri}(q^{-1})\} \cdot \prod_{j=1, j \neq i}^n den\{H_{Rj}(q^{-1})\} \right)}{\prod_{i=1}^n den\{H_{Ri}(q^{-1})\}} \quad (7)$$

Even if this model may seem too complicated, it has the benefit that imposing such a structure to the compensator leads to a finite output signal of the compensator. Therefore, it does not overload the command of the original system and can be used in various conditions.

Knowing the model of the non-compensated multimodal system and the reference model, the compensator design becomes trivial. The compensator's transfer function can be obtained as the simple fraction in Eq. (8).

$$H_C(q^{-1}) = \frac{H_R(q^{-1})}{H(q^{-1})} \quad (8)$$

Or, in a more detailed form as shown in Eq. (9). Although it looks complicated, this version can easily be implemented on a numerical computer using convolution products and polynomial sums.

$$H_C(q^{-1}) = \frac{\sum_{i=1}^n \left( num\{H_{Ri}(q^{-1})\} \cdot \prod_{j=1, j \neq i}^n den\{H_{Rj}(q^{-1})\} \right) \cdot \prod_{i=1}^n den\{H_i(q^{-1})\}}{\sum_{i=1}^n \left( num\{H_i(q^{-1})\} \cdot \prod_{j=1, j \neq i}^n den\{H_j(q^{-1})\} \right) \cdot \prod_{i=1}^n den\{H_{Ri}(q^{-1})\}} \quad (9)$$

The overall behavior of the system is obtained by composing the compensator's transfer function with the real non-compensated system's transfer function  $H'(q^{-1})$ . The resulting system has the transfer function shown in Eq. (10).

$$H_0(q^{-1}) = H_c(q^{-1}) \cdot H'(q^{-1}) \quad (10)$$

It is important to note here that the real non-compensated system's transfer function may differ from the one used in the compensator's design due to errors in the system's identification. This difference may lead to some residual vibrations in the compensated system's output. Thus, it may alter the performance of the vibrations suppression. A better solution requires dynamically changing the parameters of the compensator to minimize the error. However, this aspect will not be covered in this paper.

#### 4. SINGLE MODE VIBRATION SUPPRESSION SIMULATION

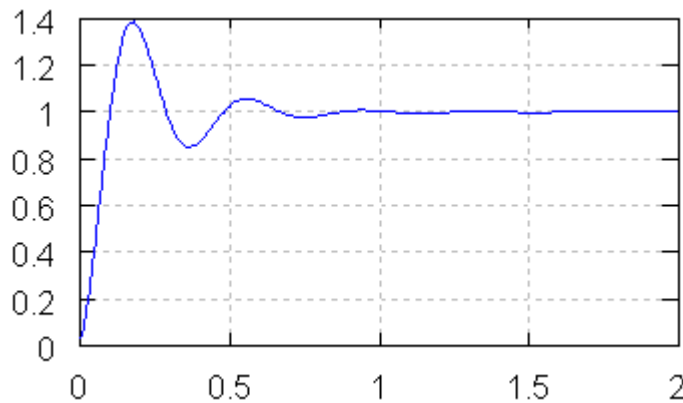
In the first example, the non-compensated system is a singlemode structure. The modal behavior can be described by the following parameters.

$$\begin{cases} K = 1 \\ f_n = 2.8\text{Hz} \\ \xi = 0.21 \end{cases} \quad (11)$$

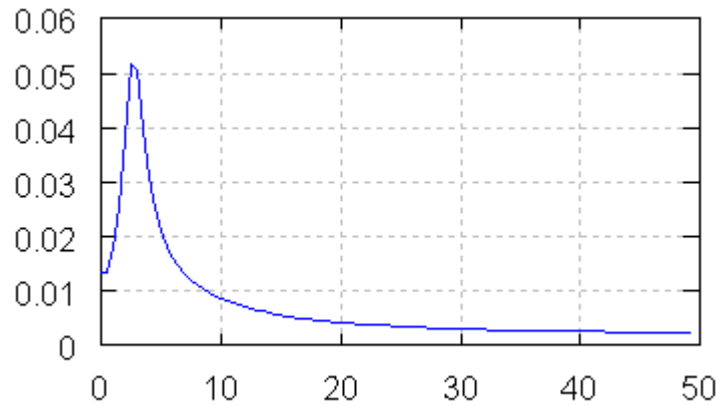
The identified model used in the compensator design contains a +10% frequency identification error and a -5% damping ratio identification error, therefore its parameters are given below.

$$\begin{cases} K = 1 \\ f_n = 3.08\text{Hz} \\ \xi = 0.1932 \end{cases} \quad (12)$$

A unit step test signal is applied to the real system in order to have a visual inspection of the output. The resulting time and frequency graphics are shown in Fig. 2 and Fig. 3. All the simulations in this paper have been made using the open source numerical simulation software Octave.org described in [2].



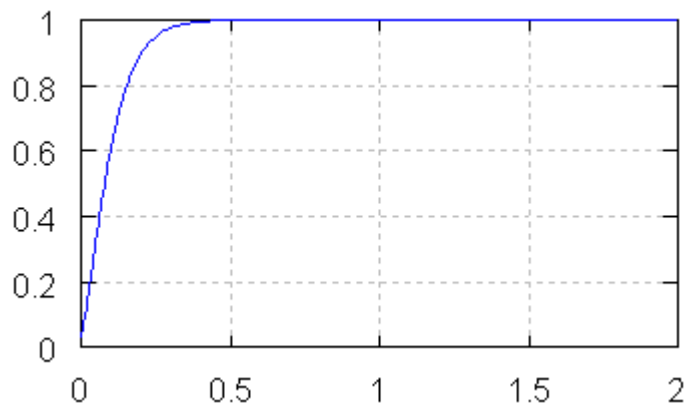
**Fig. 2 – Real non-compensated system step response (time)**



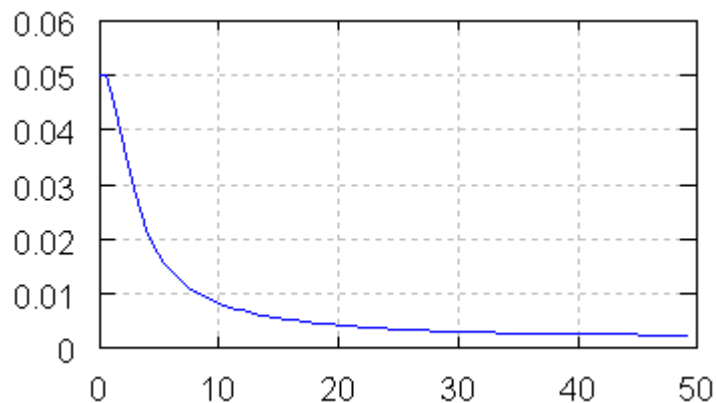
**Fig. 3 – Real non-compensated system response spectrum (frequency)**

The numerical facts show an overshoot of 38.624% and a settling time of 0.78 seconds. The identified system however has slightly different values for these performance criteria but they are of little interest at this moment.

After having the identified transfer function, the next step is obtaining the suitable reference model. As indicated in the previous section in Eq. (6), the reference model has the same structure as the identified system, except for the damping ratio that is 1. The simulation of the reference model to unit step reveals the behavior in Fig. 4 and Fig. 5.



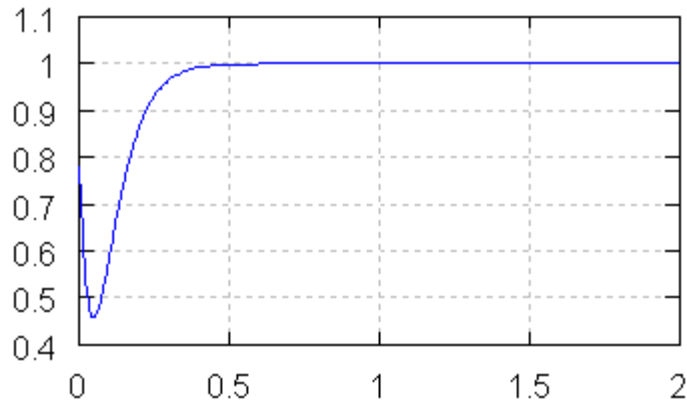
**Fig. 4 – Reference model step response (time)**



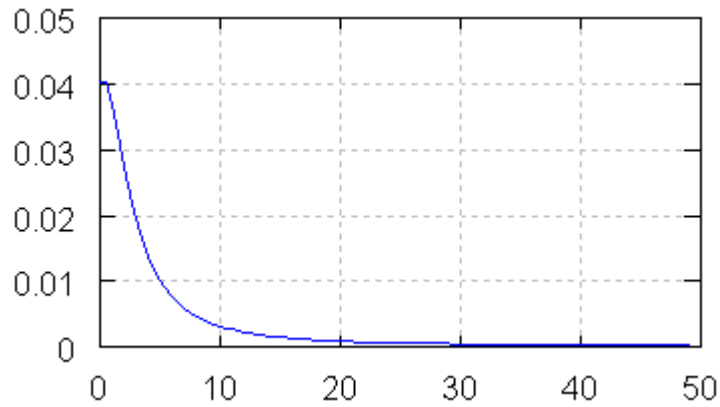
**Fig. 5 – Reference model response spectrum (frequency)**

The last step of the design is calculating the compensator transfer function from the reference model and the identified model as in Eq. (8). The resulting transfer function is given in Eq. (13) and the graphical output of the unit step simulation in Fig. 6 and Fig. 7.

$$H_c(q^{-1}) = \frac{0.041654 - 0.077702 \cdot q^{-1} + 0.037451 \cdot q^{-2}}{0.053348 - 0.089387 \cdot q^{-1} + 0.037451 \cdot q^{-2}} \quad (13)$$

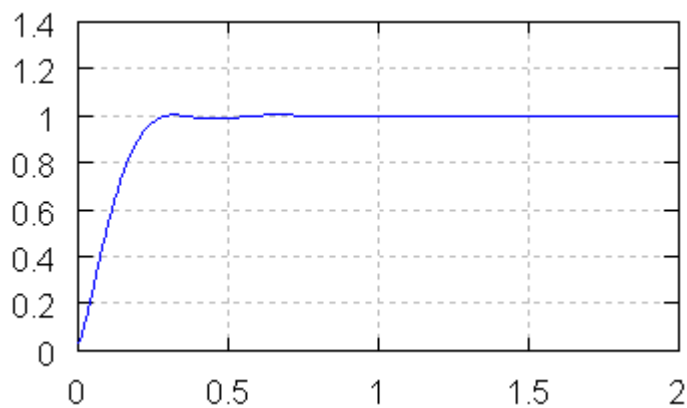


**Fig. 6 – Compensator step response (time)**

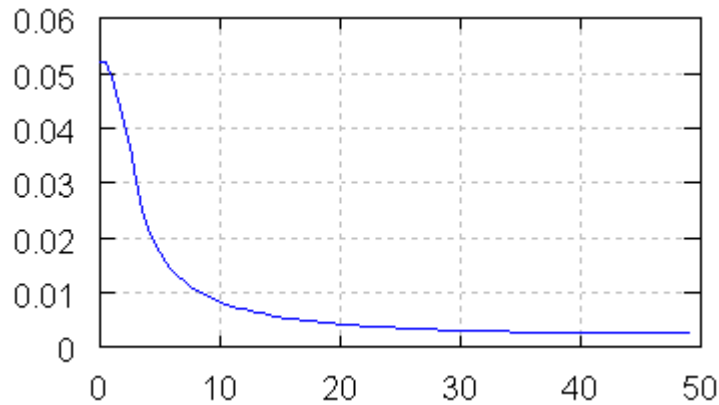


**Fig. 7 – Compensator response spectrum (frequency)**

Finally, the compensator can be added to the real system's input to reshape the original reference signal. The resulting step response is shown in Fig. 8 and Fig. 9.



**Fig. 8 – Compensated system step response (time)**



**Fig. 9 – Compensated system response spectrum (frequency)**

A noticeable aspect of the identification error is the presence of a vague residual vibration in the system's output. The compensated system's overshoot is of 0.41% and the settling time is 0.26 seconds.

The overall improvements in the system's performance is of 98.93% for the overshoot and of 66.67% for the settling time. In other words, the compensated system is much more accurate than the original system and also is much faster.

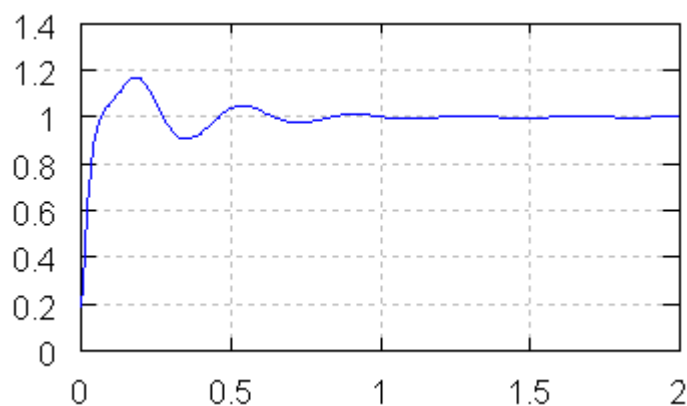
### 5. THE 5-DOF VIBRATIONS SUPPRESSION SIMULATION

Most of the real systems are multimodal, therefore an example of such a system is presented in this section. A 5-DOF system has the parameters given in Table 1.

**Table 1 – Real system modal parameters**

Gain, $K_i$	Natural frequency, $f_{ni}$ [Hz]	Damping ratio, $\xi_i$
0.35	2.8	0.12
0.32	7.1	0.15
0.18	11.3	0.11
0.10	16.4	0.20
0.05	19.1	0.13

The identified system parameters differ from the real ones with a coefficient of +5% for frequencies and -5% for damping ratios.



**Fig. 10 – Real system step response (time)**

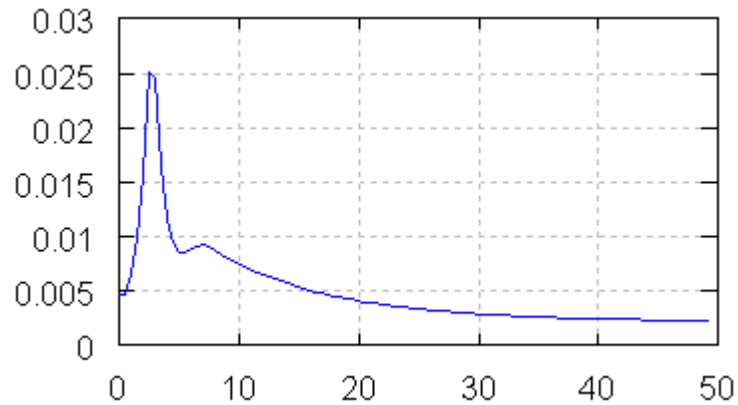


Fig. 11 – Real system response spectrum (frequency)

The real system's overshoot is 16.907% and the settling time is 0.77 seconds. Again, according to the second section of this paper, the reference model must contain the same modal structure and parameters, except the damping ratio which is always 1. The reference model's response to unit step is shown in Fig. 12 and Fig. 13.

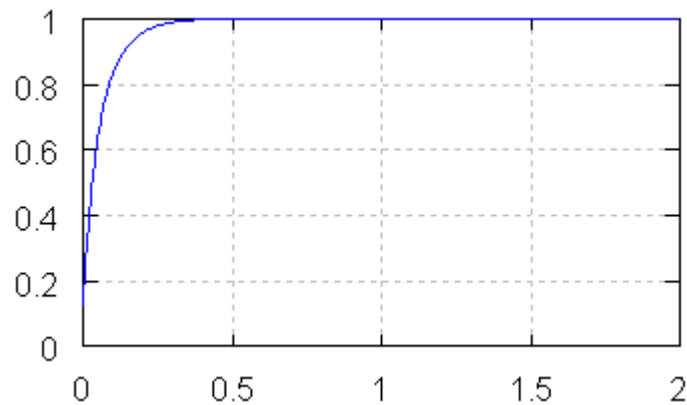


Fig. 12 – Reference model step response (time)

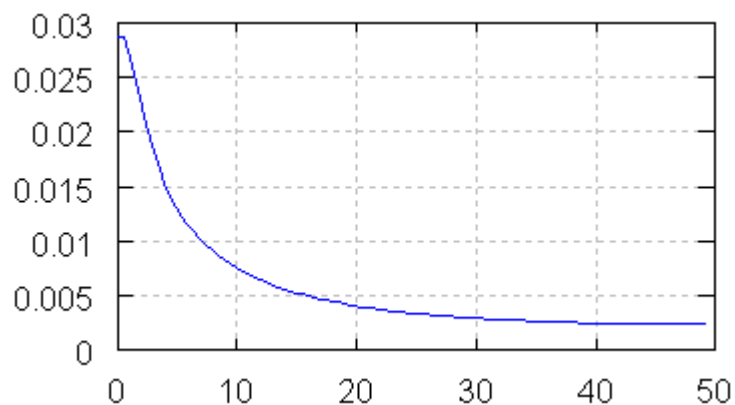
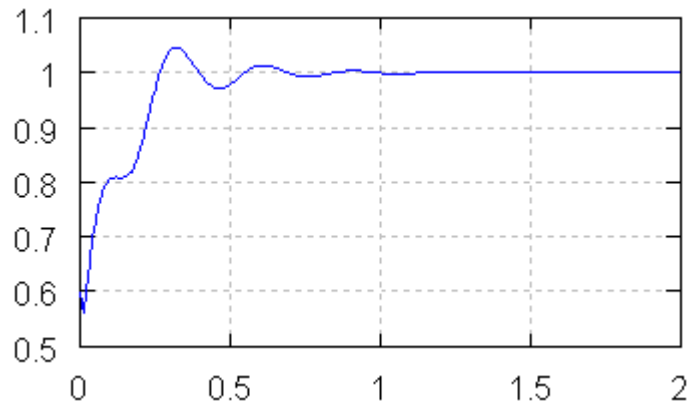
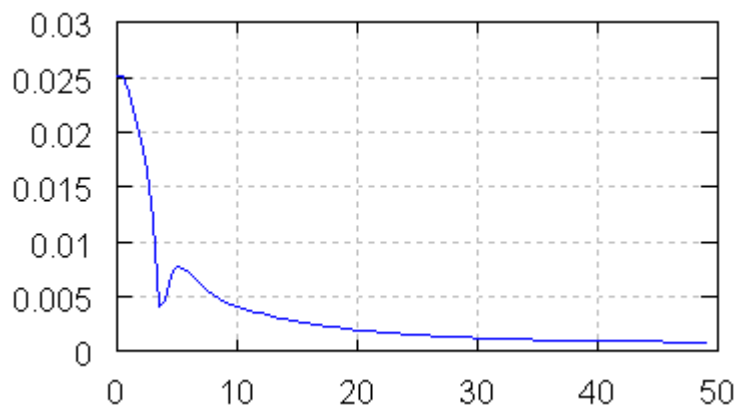


Fig. 13 – Reference model response spectrum (frequency)

According to Eq. (8), the compensator is obtained from the reference model and the identified model. The simulation of the compensator revealed the behavior in Fig. 14 and Fig. 15. The actual transfer function is rather complicated and was not included in this paper, but pose no problem in a digital implementation.

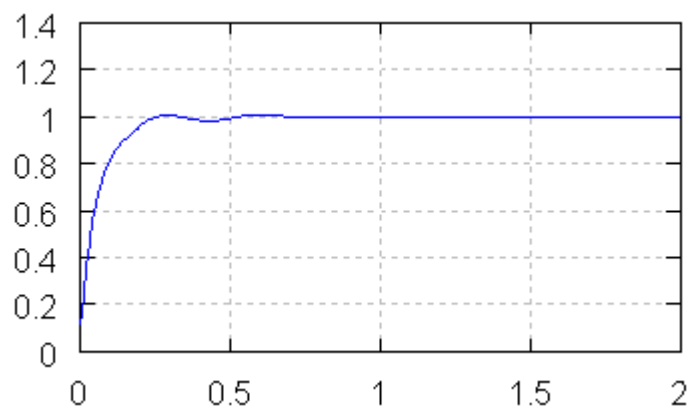


**Fig. 14 – Compensator step response (time)**



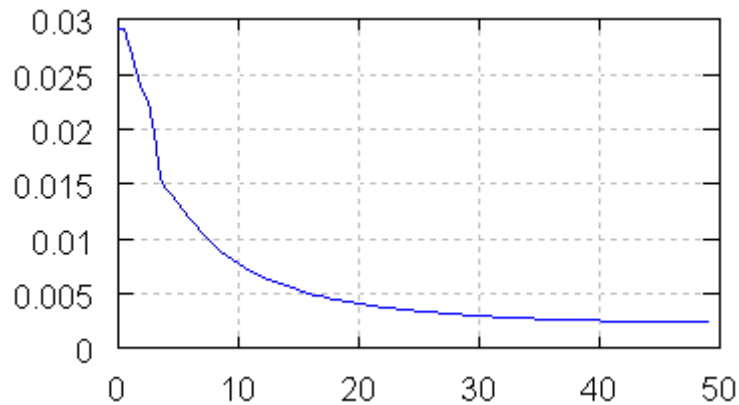
**Fig. 15 – Compensator response spectrum (frequency)**

Connecting the compensator and the original system, and taking into account that the original model is not perfectly identifiable, the result of the compensated system's simulation is shown in Fig. 16 and Fig. 17.



**Fig. 16 – Compensated system step response (time)**

The resulting overshoot is 0.928% and the settling time is 0.23 seconds. This means an improvement of overshoot with 94.50% and an improvement of the settling time of 70.12%, therefore a faster and more accurate system.



**Fig. 17 – Compensated system step response (time)**

## 6. CONCLUSIONS

An open loop non-invasive technique such as the one presented in this paper allows dramatically improving the performance of the mechatronic systems via flexible robust software techniques of digital control. The resulting compensated systems are faster and more accurate.

The biggest disadvantage of this method is the imperfection of the identification and the therefore the sensitivity to parametric errors. A dynamic online adjustment of the compensator / identified model parameters may improve the behavior of the compensator. However, the robustness of the presented method may prove sufficient for the majority of applications.

## 7. ACKNOWLEDGEMENT

This paper is the result of a research partially financed by the “Petru Maior” University PhD research grant contract No. 1520 / 18.05.2009.

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