CONSIDERATIONS ABOUT THE VIBRATIONS OF THE CAM – FOLLOWER SYSTEM

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Abstract**.** In the present paper, it is considered vibrating system acting by a rotation cam, which has a constant angular velocity and a motion law saw-tooth type. Using the Lagrange formalism or the Newton one, the motion mathematical model of the valve is obtained. For a numerical application, the motion differential equation is integrated with the help of integral Laplace transform. Thus, results the motion law of the valve under the form of a time function. The determination is made in two evaluation system in order to obtain a bigger accuracy of the results.

1.MATHEMATICAL MODEL OF MOTION

It is considered the vibrating system from fig 1.a, system which is energized by a cam. One end of the elastic resort k_2 represents a follower, being driven by a cam which has a constant angular speed ω and a motion law saw-tooth type with a maximum displacement h (fig. 1.b). In this figure the temporary variable t is defined as x, this definition being used from here on.
Using the Lagrange formalism or the Newton one, the mathematical model is obtained in case of a valve with m mass:

$$m\ddot{y}(t)+c\dot{y}(t)+k_1y(t)+k_2\left[y(t)-y_1(t)\right]=0 \qquad (1)$$

or:

$$\ddot{y}(t)+2n\dot{y}(t)+\omega_n^2y(t)=\varpi_n^2y_1(t), \qquad (1^{(1)})$$

where:

$$k_e=k_1+k_2,\ \frac{c}{m}=2n,\ \frac{k_e}{m}=\omega_n^2,\ \frac{k_2}{m}=\varpi_n^2,$$

c being the dumping constant.
The motion law for the cam for a T time period is:

$$y_1(t)=\frac{h}{T}t,\ 0<t<T, \qquad (2)$$

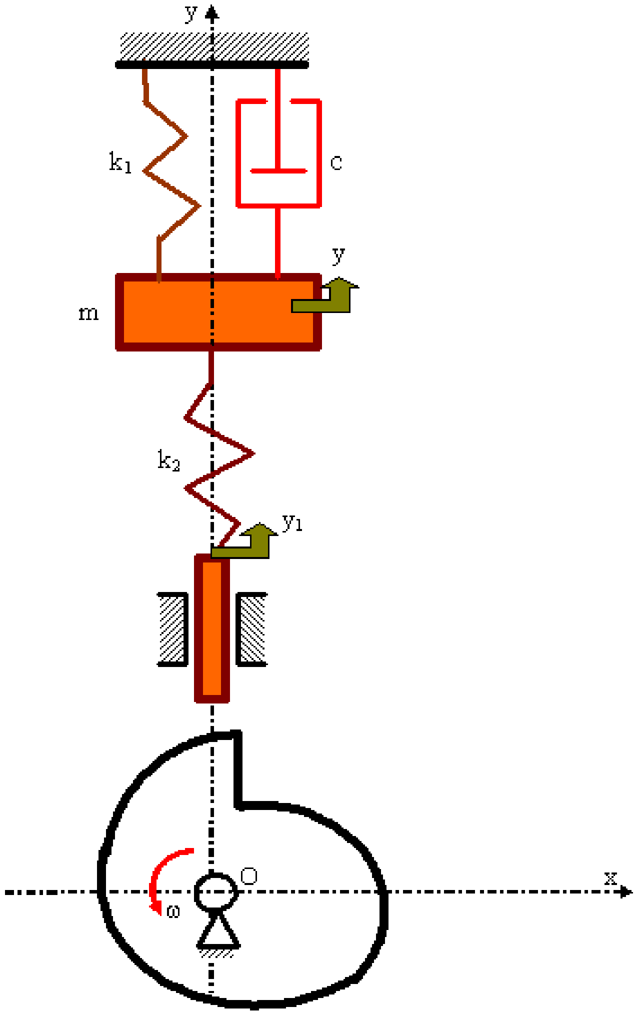
thus, the mathematical model ($1^{(1)}$) becomes:

$$\ddot{y}(t)+2n\dot{y}(t)+\omega_n^2y(t)=\varpi_n^2\frac{h}{T}t. \qquad (1^{(2)})$$

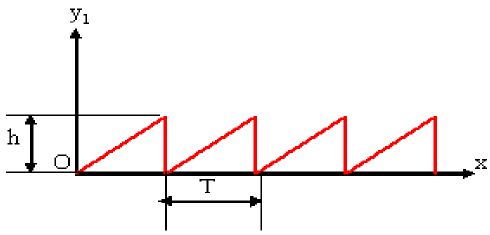
Let's consider a definite case with: $m=1[Kg]$, $h=0{,}05[m]$, $T=1[s]$, $k_1=300\left[\frac{N}{m}\right]$, $k_2=20\left[\frac{N}{m}\right]$, $c=15\left[Nsm^{-1}\right]$.

Thus the equation ($1^{(2)}$) becomes:

$$\ddot{y}(t)+15\dot{y}(t)+320y(t)=t \qquad (1^{(3)})$$



a.



b.

Fig. 1 – Mechanical model of the vibrating system

2. DYNAMIC RESPONSE

Applying the unilateral Laplace transform to the equation ($1^{(3)}$), with respect to time and with the initial conditions $y(0)=0$, $\dot{y}(0)=1\left[\frac{m}{s}\right]$, it results the equation:

$$s^4\tilde{y}(s) - s^2 + 15s^3\tilde{y}(s) + 320s^2\tilde{y}(s) = 1,$$

with the result:

$$\tilde{y}(s) = \frac{s^2+1}{s^2\left(s^2+15s+320\right)} \qquad (3)$$

Using the mathematical software *Mathematica*, from (3) it results the solution of the equation ($1^{(3)}$) as a time function:

$$y(t) = \frac{1}{21606400}\cdot$$
$$\cdot\left\{-3165 + 67520t + e^{-\frac{15}{2}t}\left[3165\cos\left(\frac{\sqrt{1055}}{2}t\right) + 40877\sqrt{1055}\sin\left(\frac{\sqrt{1055}}{2}t\right)\right]\right\},$$

its representation being shown in fig. 2.
Using *Maple V* software, the time function is obtained.

$$y(t) = e^{-7,5t}\left[0,6186\cdot 10^{-5}\cos(21,0653t) + 0,0047\sin(21,0653t)\right] +$$
$$+e^{-0,3921\cdot 10^{-10}t}\left[0,000064\sin(3.1415t) - 0,6186\cdot 10^{-5}\cos(3.1415t)\right],$$

its representation shown in figure 3.

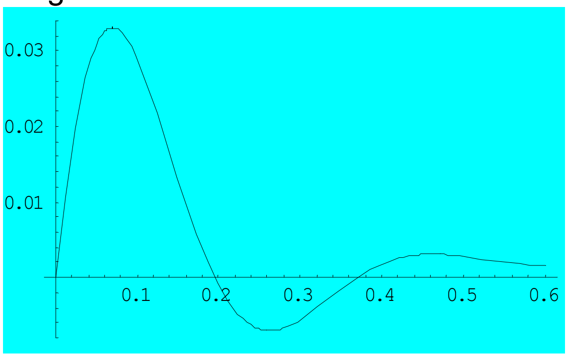


Fig.2 y=y(t)

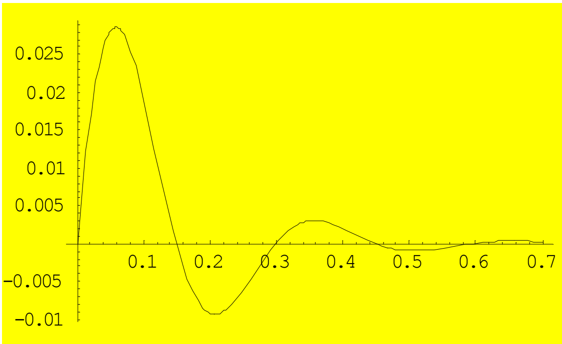


Fig. 3 y =y(t)

Only the first harmonic is shown since for the numerical values chosen, the variation of the function $y = y(t)$ based on the numbers of harmonics is not representative.

For a more accurate result of the calculus, it is necessary to use several mathematical softwares. This way, the results can be easily compares as in the above case.

3. CAM LAW MOTION REPRESENTATION

The function $y_1 = y_1(t)$, defined by (2), is rewritten using the Fourier series:

$$y_1(t) = \sum_{i=1}^{n} a_i \sin(i\pi t), \quad (4)$$

where:

a_1=0,03183, a_2=-0,01591, a_3=0,01061, a_4=-0,00795, a_5=0,00636, a_6=0,005305, ...,
only the sinus function terms being shown; the function is obtained using *Maple V* software.

The above pictures present this function, the difference being made by the number of terms that are taken into consideration.

More terms taken into consideration, results in a high accuracy of the graphical representation of $y_1 = y_1(t)$ function.

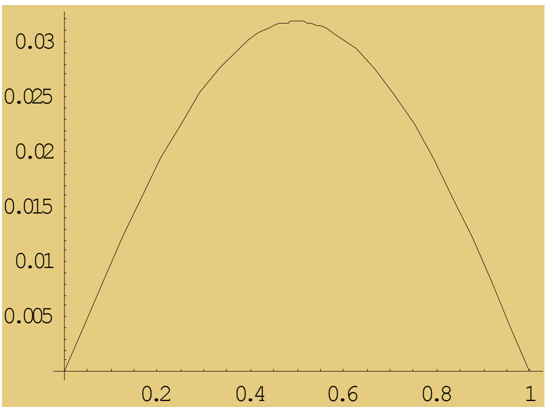


Fig. 4 First harmonic representation

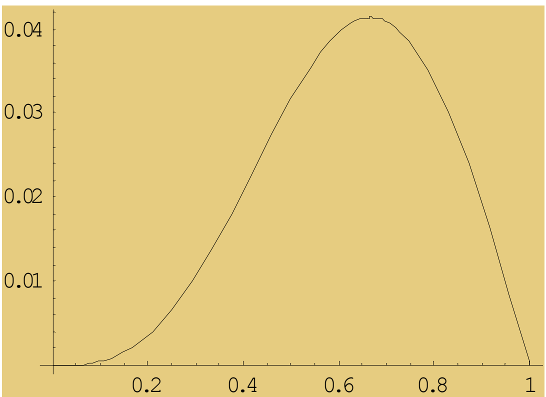
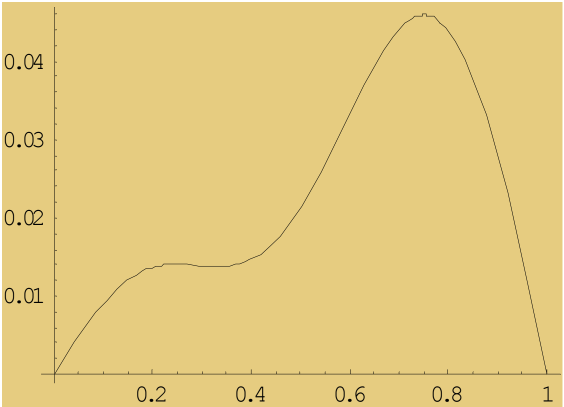


Fig. 5 First two harmonics representation



a. T=1 [s] ***b. T=3 [s]***

Fig. 6 First three harmonics representation

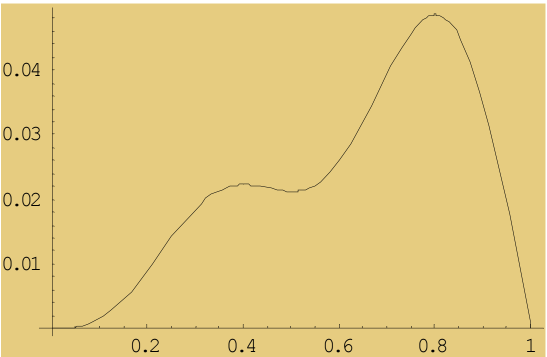


Fig. 7 First four harmonics representation

$$y_1(t) = \sum_{i=1}^{4} a_i \sin(i\pi t)$$

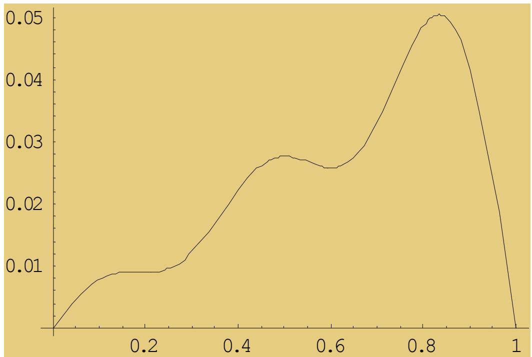
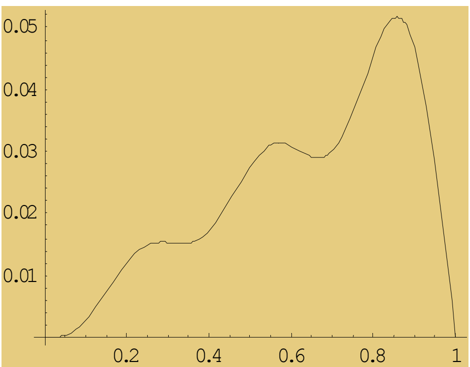
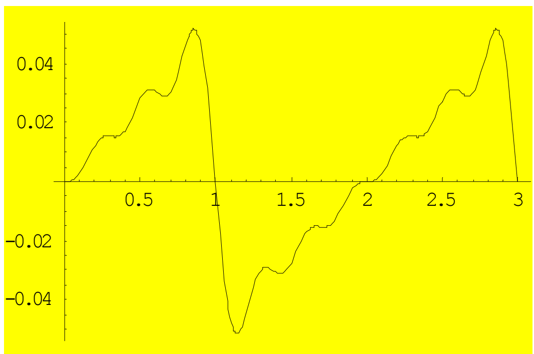


Fig. 8 First five harmonics representation

$$y_1(t) = \sum_{i=1}^{5} a_i \sin(i\pi t)$$



a. T=1 [s]



b. T=3 [s]

Fig. 9 First six harmonics representation

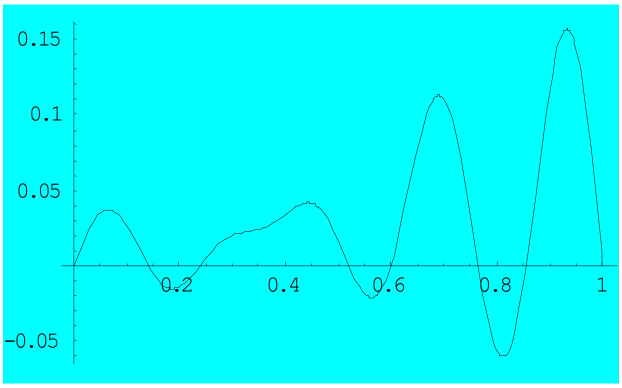
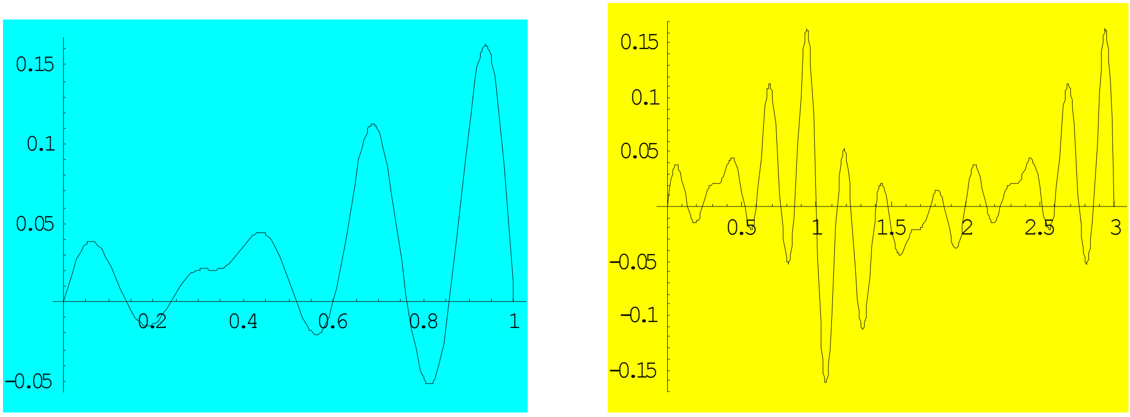


Fig. 10 First ten harmonics representation

$$y_1(t)=\sum_{i=1}^{10} a_i \sin(i\pi t)$$

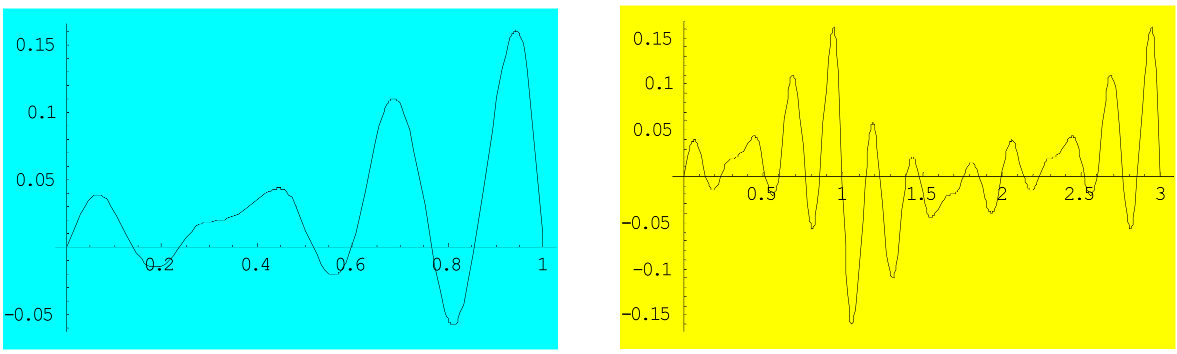


a. T=1 [s]

b. T=3 [s]

Fig. 11 First fifteen harmonics representation

$$y_1(t)=\sum_{i=1}^{15} a_i \sin(i\pi t)$$



a. T=1 [s]

b. T=3 [s]

Fig. 12 First twenty harmonics representation

$$y_1(t) = \sum_{i=1}^{20} a_i \sin(i\pi t).$$

4. CONCLUSIONS

One can easily notice that, even from the representation of the first fifteen harmonics, the variation of this dependency with the number of used harmonics is no longer significant. However, in a definite case as that of an internal combustion engine valves, all harmonics up to 150 ranges are to be taken into consideration.

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