MATHEMATICAL MODEL CALCULATING THE FORCES ACTING FOR BEARINGS

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ABSTRACT: The geometric parameters for bearings with cylindrical rolls and rings with two shoulders are: R_s - exterior sphere maximum radius;X_s - distance from the center of the maximum exterior sphere to the center of running C_r:μ^o - contact angles of the ring's shoulders;R_{br} - swell radius for the roll;R_{ble} - swell radius for the exterior ring;R_{bli} - swell radius for the interior ring;J_b - initial lost motion between the ring and the roll;J_e - initial lost motion between the roll and the ring's shoulder;L_r - length of the rectilinear generator of the running track;L_e - length of the real track.

1. DETERMINING THE ROLLING ELEMENT'S BALANCE

To determine the rolling elements balance we will have to calculate in the radial field compared to the S_3 system. In this field, the exterior ring's shifts are null, and those of the interior ring are expressed in C_r, depending on the relative shifts of the N_2 joint in correlation with the N_1 joint, in the S_1 system by:

$$\left\{\vec{u}\right\} = \begin{Bmatrix} u_r \\ u_\theta \\ u_z \end{Bmatrix} = [G_2]\left(\left\{\overrightarrow{dep}(N_2)\right\}_{S_1} - \left\{\overrightarrow{dep}(N_1)\right\}_{S_1}\right) \quad (1.1)$$

The rolling elements balance is determined by a Newton-Raphson iteration loop for the rolling element center shift vector:

$$\left\{\vec{v}\right\} = \begin{Bmatrix} u_r \\ u_\theta \\ u_z \end{Bmatrix} \quad (1.2)$$

For a given vector $\{\vec{v}\}$ and using the hypothesis the rolling elements and ring sections cannot be deformed, as the bearing geometry, we can determine the contact points and the solid bodies shift. When this shift is greater than the lost motion, the resulting forces and moments in C_r can be determined. The rolling elements are exposed to contact forces in each ring, and surplus forces are expressed trough:

$$\left\{\vec{R}\right\} = -\sum F_{ext} = -\begin{Bmatrix} F_r^{I_e}(v_r, v_\theta, v_z) \\ F_z^{I_e}(v_r, v_\theta, v_z) \\ M_n^{I_e}(v_r, v_\theta, v_z) \end{Bmatrix} - \begin{Bmatrix} F_r^{I_i}(v_r, v_\theta, v_z) \\ F_z^{I_i}(v_r, v_\theta, v_z) \\ M_n^{I_i}(v_r, v_\theta, v_z) \end{Bmatrix} \quad (1.3)$$

The Newton-Raphson method consists in determining after each iteration, the $\left\{\Delta\vec{v}\right\}$ vector as well as the surplus forces vector calculated so that the shifts $\left(\left\{\vec{v}\right\} + \left\{\Delta\vec{v}\right\}\right)$ make a null vector. For each iteration the $\left\{\Delta\vec{v}\right\}$ vector is evaluated depending on $\left\{\vec{v}\right\}$ and $\left\{\vec{R}\right\}$ through the linear systems:

$$\left[\frac{\partial\left\{\vec{R}\right\}}{\partial\left\{\vec{v}\right\}}\right]_{\vec{v}_i} \left\{\Delta\vec{v}\right\} = [K_t]\left\{\Delta\vec{v}\right\} = \left\{\vec{R}\right\} \quad (1.4)$$

where [K_i] is the tangential rigidity matrix, associated to the "rolling elements-rings" system, calculated in the C_r point. It is expressed as the sum of the tangential rigidity matrixes associated to the rolling elements contacts with the interior and exterior ring:

$$\left[K_t\right]_i = \left[-\frac{\partial\left\{\vec{F}^{I_e}\right\}}{\partial\left\{\vec{v}\right\}} \right]_{\vec{v}_i} + \left[-\frac{\partial\left\{\vec{F}^{I_i}\right\}}{\partial\left\{\vec{v}\right\}} \right]_{\vec{v}_i} = \left[K_t^{I_e}\right]_{\vec{v}_i} + \left[K_t^{I_i}\right]_{\vec{v}_i} \tag{1.5}$$

The rolling elements balance is obtained when:

$$\left\|\Delta\vec{v}\right\| \le \varepsilon_v \qquad \left\|\vec{R}\right\| \le \varepsilon_r$$

Rolling elements loading characteristic data: the forces at the bearing shoulders tracks level, oscillation or contact angles are known.

2. TANGENTIAL RIGIDITY MATRIX CALCULATION BETWEEN JOINTS

Previously defined rigidity matrixes $\left[K_t^{I_e}\right]$ and $\left[K_t^{I_i}\right]$ calculation is carried out in the S_1 system. They tie the radial field shifts of the center C_r of the rolling elements to the C_r points of every ring. An equivalent rigidity matrix can be defined between the C_r points of every ring:

$$\left[\overline{K}\right] = \begin{bmatrix} K & -K \\ -K & K \end{bmatrix}_{6x6} \tag{1.6}$$

using the formula:

$$\left[K\right]_{3x3} = \left[K_t^{I_e}\right]\left[K_t^{I_e} + K_t^{I_i}\right]^{-1}\left[K_t^{I_i}\right] \tag{1.7}$$

The use of the above defined geometric matrix allows the expression of the $\left[\overline{K}\right]$ matrix in a „*3D*" - S_1 reference system and the definition of the elementary rigidity matrix associated with the rolling elements between the N_1 and N_2 joints:

$$\underbrace{\left[K_t^e\right]_{S_1}}_{12x12} = {}^T\left[G\right]\left[\overline{K}\right]\left[G\right] \tag{1.8}$$

where:

$$\left[G\right] = \underbrace{\begin{bmatrix} G_1 \ldots & 0 \\ \vdots & \vdots \\ 0 \ldots & G_2 \end{bmatrix}}_{6x12} \tag{1.9}$$

This is the global matrix associated to the rolling elements matrixes and to the rest of the structure before solving the non-linearity problem.

3. FORCES APPLIED BY ROLLING ELEMENTS ON JOINTS

In the same way, the forces applied by the rolling elements on the N_1 şi N_2 joints can be calculated based on the forces applied on the rings in the radial field and evaluated in the C_r point:

$$\underbrace{\left\{F^{N_1}\right\}_{S_1}}_{6x1} = {}^T\underbrace{\left[G_1\right]}_{6x3}\underbrace{\left\{-F^{I_e}\right\}}_{3x1}$$

$$\underbrace{\left\{F^{N_2}\right\}_{S_1}}_{6x1} = {}^T \underbrace{\left[G_2\right]}_{6x3} \underbrace{\left\{-F^{I_i}\right\}}_{3x1} \qquad (1.10)$$

These elementary force vectors are assembled with additional force vectors of the general non-linearity problem.

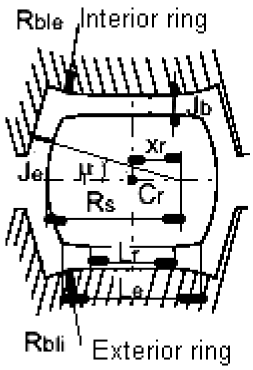

Figure 1. Bearing geometric parameters

The geometric parameters for bearings with cylindrical rolls and rings with two shoulders are:

- R_s - exterior sphere maximum radius;
- X_s - distance from the center of the maximum exterior sphere to the center of running C_r:
- μ^o - contact angles of the ring's shoulders;
- R_{br} - swell radius for the roll;
- R_{ble} - swell radius for the exterior ring;
- R_{bli} - swell radius for the interior ring;
- J_b - initial lost motion between the ring and the roll;
- J_e - initial lost motion between the roll and the ring's shoulder;
- L_r - length of the rectilinear generator of the running track;
- L_e - length of the real track.

In the reference system S_3 tied to the exterior ring and for the given $\left\{\vec{u}\right\}$ and $\left\{\vec{v}\right\}$ shifts, the rolls and interior ring oscillate. We consider the aimed angle A_e between the exterior ring's axis and the bearing's axis, and the aimed angle A_i between the interior ring's axis and the roll's axis, that will be defined depending on the $\left\{\vec{u}\right\}$ and $\left\{\vec{v}\right\}$ shifts components:

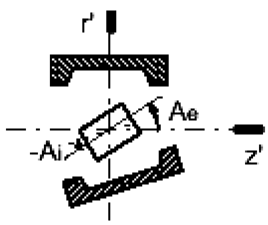

Figure 2. Shifts between rolls and rings

In a similar way we can define the μ_i angle associated to each shoulder of the exterior and interior rings, as the actual angle between the normal in the contact point and the roll's axis.

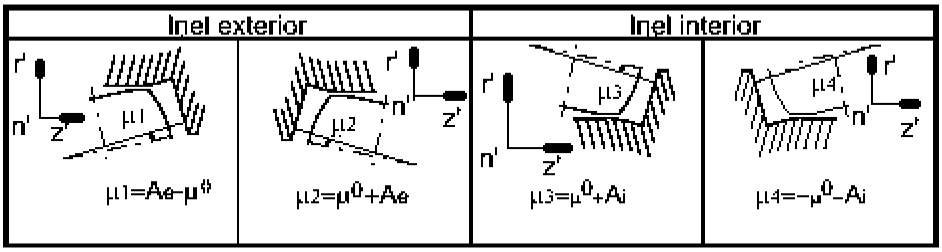

Figure 3. Ring shoulders angles

4. CALCULATING THE FORCES ACTING BETWEEN THE RUNNING TRACKS AND THE ROLLS

Only the forces on the r' direction in the S_3 system will be calculated. On this direction the δ_i and δ_e shifts of the roll in relation to each running track are expressed by:

$$\delta_i = u_r - v_r - J_b \qquad \delta_e = v_r - J_b \tag{1.11}$$

When these values are positive, there is contact between the roll and the bearing running track. Calculating the corresponding forces is done by into N potential contact zones the rolls and running tracks. This meshing is done symmetrically, in relation with the C_r center of the rolls:

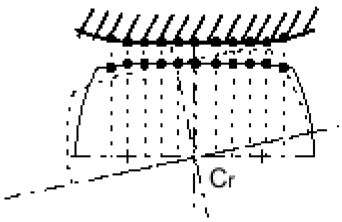

Figure 4. Meshing in relation with the center of the roll

For narrow and frequent samples d_e, expressed by:

$$d_e = \frac{L_e}{N}$$

situated around a joint k of z_k abscissa the squash between the two bodies is expressed taking into account the α^1 relative oscillation angle of the two bodies and possibly even the curvature radiuses, using the formula:

$$\Delta d = \delta + \alpha\, z_k - h_k^1 - h_k^2 \tag{1.12}$$

where h_k^i is a term that allows taking into account possible curvatures R_{ci} of the body i. If the k point is outside the rectilinear generator, it means that:

$$\left|z_k\right| \geq \frac{L_r}{2}$$

and we have a curvature, the term h_k^i is expressed by:

$$h_k^i = R_{ci} - \sqrt{R_{ci} - \left(z_k - \frac{L_r}{2}\right)^2} \tag{1.13}$$

and in other cases it is null.

Contact forces are expressed trough Hertz's theory for narrow domains:

$$Q_k = C_f \left(\Delta d\right)^{\frac{10}{9}} d_e \tag{1.14}$$

These forces generate a moment in C_r:

$$M_k = Q_k z_k \tag{1.15}$$

The forces and moments that are use by the running track on the roll will be:

$$Q_I = \sum_{k=1}^{N} Q_k \qquad M_I = \sum_{k=1}^{N} M_k \tag{1.16}$$

In conclusion, $Q_{Ie}(v_r, v_\theta, v_z)$ and $M_{Ie}(v_r, v_\theta, v_z)$ are the forces and moments that act upon the rolls in the C_e point due to the exterior ring, and $Q_{Ii}(v_r, v_\theta, v_z)$ and $M_{Ii}(v_r, v_\theta, v_z)$ due to the interior ring.

5. CALCULATING THE FORCES IN THE RING'S SHOULDERS

The actual directions of contact between the roll's extremities and the ring's shoulders were presented in figure 1.8. Along these directions the lost motion between the roll and the *i* shoulder is expressed depending on the initial lost motion J_e, on the hollow contact angle μ^o, on the under load contact angle μ_i and on the distance X_s by:

$$\widetilde{J}_i = J_e + X_s(\cos\mu_i - \cos\mu^o) \tag{1.17}$$

Approaches between the shoulders and the roll along the contact direction are expressed by:

$$\begin{aligned}
\delta_1 &= v_z \cos\mu^o + v_r \sin\mu^o - \widetilde{J}_1 \\
\delta_2 &= -v_z \cos\mu^o + v_r \sin\mu^o - \widetilde{J}_2 \\
\delta_3 &= -(u_z - v_z)\cos(\mu^o - u_\theta) + (u_r - v_r \sin(\mu^o - v_\theta) - \widetilde{J}_3 \\
\delta_4 &= (u_z - v_z)\cos(\mu^o - u_\theta) + (u_r - v_r)\sin(\mu^o - v_\theta) - \widetilde{J}_4
\end{aligned} \tag{1.18}$$

When δ_i is positive, there is contact between the roll and the *i* shoulder and the resulting effort Q_{Ei} directed along the norm in contact is expressed by Hertz's law for punctual contact:

$$Q_{Ei}(v_r, v_\theta, v_z) = C_{ep}(\delta_i)^{3/2} \tag{1.19}$$

and the resulting moment in C_r by:

$$M_{Ei} = -Q_{Ei} X_S \sin\mu_i \tag{1.20}$$

The balance equations and the Jacobian matrixes of the roll-ring system are, in the S_3 system, taking the F_c centrifugal force into account and the M_g gyroscopic moment that act upon the roll:

$$F_r(v_r, v_\theta, v_z) = \underbrace{-Q_{Ie} - Q_{E2}\sin\mu^o - Q_{E1}\sin\mu^o}_{Ie/rola} + \underbrace{Q_{Ii} - Q_{E4}\sin(\mu^o + u_\theta) - Q_{E3}\sin(\mu^o - u_\theta) + F_c}_{Ii/rola} =$$
$$= F_r^{Ie}(v_r + v_\theta + v_z) + F_r^{Ii}(v_r + v_\theta + v_z) + F_c \tag{1.21}$$

$$F_z(v_r, v_\theta, v_z) = \underbrace{Q_{E2}\cos\mu^o - Q_{E1}\cos\mu^o}_{Ie/rola} + \underbrace{Q_{E4}\cos(\mu^o + u_\theta) - Q_{E3}\cos(\mu^o - u_\theta)}_{Ii/rola} =$$
$$= F_r^{Ie}(v_r + v_\theta + v_z) + F_r^{Ii}(v_r + v_\theta + v_z) \tag{1.22}$$

$$M_\theta(v_r, v_\theta, v_z) = \underbrace{-M_{Ie} - M_{E2} - M_{E1}}_{Ie/rola} + \underbrace{M_{Ii} - M_{E4} - M_{E3}}_{Ii/rola} =$$
$$= M_\theta^{Ie}(v_r + v_\theta + v_z) + M_\theta^{Ii}(v_r + v_\theta + v_z) + M_g \tag{1.23}$$

The unknowns of the problem are v_r, v_θ, v_z. The tangential rigidity matrixes associated to the contacts between the exterior ring and the roll and to the contacts between the interior ring and the roll are the Jacobian matrixes associated to the forces-shifts relations, expressed trough:

$$\left[K_t^{Ie}\right]=-\begin{bmatrix} \frac{\partial F_r^{Ie}}{\partial v_r} & \frac{\partial F_r^{Ie}}{\partial v_\theta} & \frac{\partial F_r^{Ie}}{\partial v_z} \\ \frac{\partial M_\theta^{Ie}}{\partial v_r} & \frac{\partial M_\theta^{Ie}}{\partial v_\theta} & \frac{\partial M_\theta^{Ie}}{\partial v_z} \\ \frac{\partial F_z^{Ie}}{\partial v_r} & \frac{\partial F_z^{Ie}}{\partial v_\theta} & \frac{\partial F_z^{Ie}}{\partial v_z} \end{bmatrix} \qquad \left[K_t^{Ii}\right]=-\begin{bmatrix} \frac{\partial F_r^{Ii}}{\partial v_r} & \frac{\partial F_r^{Ii}}{\partial v_\theta} & \frac{\partial F_r^{Ii}}{\partial v_z} \\ \frac{\partial M_\theta^{Ii}}{\partial v_r} & \frac{\partial M_\theta^{Ii}}{\partial v_\theta} & \frac{\partial M_\theta^{Ii}}{\partial v_z} \\ \frac{\partial F_z^{Ii}}{\partial v_r} & \frac{\partial F_z^{Ii}}{\partial v_\theta} & \frac{\partial F_z^{Ii}}{\partial v_z} \end{bmatrix} \tag{1.24}$$

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