A QUANTITATIVE EVALUATION OF JOURNAL BEARINGS LIGHTLY LOADED

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Abstract. At present, the calculation is effected by means of numerical methods and the calculus programs are very complicated, time-consuming and, with the exception of the THD solution, rather inaccurate. The international literature of the last years contains scientific papers which bring again to the fore the analytical solution of the lubrication problem. The paper proposes a new analytical calculus model for to reaching the variation law of film pressures. This model allows for the use of dynamic boundary conditions along the rotation movement, and it can be applied to the quantitative appraisal of radial bearings with small loads.

NOMENCLATURE

c: radial clearance, (m)
e: eccentricity, (m)
h, h₀: film thickness, film thickness in which the pressure gradient is zero, (m)
L₁, L₂: unitary length, the real length of the bearing, respectively, (m)
p, p₀: film pressure, supply pressure, (Pa)
R: journal radius, (m)
Tₑ_shaft: shaft temperature, (°C)
V: runner velocity, (m/s)
W: load carrying capacity, (N)
u, v, w: velocity components, (m/s)
x, y, z: global coordinates, (m)
ε: eccentricity ratio, (nondimesional)
ϕ: attitude angle, (degree)
μ: lubricant viscosity, (Pa·s)
θ, θᵢ, θₐ: rupture angle, initial angles and final angles of the lubricant film, (degree)
ω: angular speed of the shaft, (rad/s)

1. INTRODUCTION

The differential equation of pressures proposed by Reynolds [15] on February 11, 1886, within the Royal Society of Great Britain is a milestone in the hydrodynamic lubricant theory. Although the first attempts at solving the equation were analytical, [16, 12] because of the inaccuracy of the solutions or the limited putting into practice thereof, numerical methods were used alongside the computers’ increased use, [13, 14]. Although the numerical approaches constitute a thorough spectrum of solutions as well as the necessary accuracy for finite length bearings, over the recent years a tendency toward reconsidering the analytic approach has been noted, for resolving both the hydrodynamic and even the thermodynamic issues. Mention must be made here of the papers [3, 4, 7, 8], were analytic calculus models are proposed for laminar and turbulent flow of finite bearings.

2. THEORETICAL CONSIDERATIONS

To identify the denominations, the principle schema of the radial bearing of figure (1) will be considered.
The starting point of the new model is the differential relation of pressures, proposed and thoroughly presented, [7, 8]:

\[ \frac{\partial}{\partial z} \left( \frac{h^3 \cdot \partial p}{\mu} \right) = 6 \cdot V \cdot \frac{\partial h}{\partial x} \cdot I(\theta) \]  

(1)

In this equation, \( I(\theta) \) is the correction function applied to the pressures field and has the following shape:

\[ I(\theta) = 1 - \frac{(L - L_1) \cdot e \cdot \sin \theta}{h_0 \cdot R + L \cdot e \cdot \sin \theta} \]  

(2)

Basically, the equation (1) represents the theory of short bearings, corrected by function \( I(\theta) \) with a view to fruitfully applying it within finite length bearings. The expression of function \( I(\theta) \) stemmed from a critical analysis of the theory of short bearings, and was originally deducted, [7]. The law of pressure distribution obtained through the successive integration of the differential equation, with the boundary own conditions of theory of short bearings yielded complete satisfaction in the calculus of the main parameters of bearings with medium or heavy loading, as resulted from the comparisons theory-experiment included in papers, [7, 8]. The discrepancy between theory and experiment in small load bearings is explained by the fact that the film thickness \( h_0 \) in the expression of function \( I(\theta) \) cannot be calculated unless the following boundary condition is applied: \( p = p_x, \frac{\partial p}{\partial x} = 0 \).

This condition [2, 6], give unacceptable errors in the case of small load bearings. As a conclusion, the corrected differential relation by \( I(\theta) \) function ensures a good theory-experiment concordance, regardless of the length of the radial bearing, yet sometimes leads to grave errors in the case of small load bearings.

3. RATIONALE FOR OBTAINING THE DIFFERENTIAL EQUATION OF PRESSURES

Taking into account the advantages as well as the disadvantages of the relation (1) and the differential equation in laminar regime, proposed by Reynolds:

\[ \frac{\partial}{\partial x} \left( \frac{h^3 \cdot \partial p}{12 \cdot \mu} \right) + \frac{\partial}{\partial z} \left( \frac{h^3 \cdot \partial p}{12 \cdot \mu} \right) = V \cdot \frac{\partial h}{\partial x} \]  

(3)

a new differential equation that can be analytically solved is proposed:

\[ \frac{\partial}{\partial x} \left( \frac{h^3 \cdot \partial p}{\mu} \right) = 6 \cdot V \cdot \frac{\partial h}{\partial x} \cdot k(\theta) \]  

(4)

In this case, the correction function applied to the pressures field \( k(\theta) \) has the form:
The resolving of equation (4) consists in finding the variation law of pressures and the relation of film thickness $h_0$. For that, the differential equation is integrated twice, and the boundary conditions specific to short bearings theory are applied. Integrals are solved analytically using in turn the substitution method and Sommerfeld transformations. Calculations are more time consuming than in the classical case because of function $k(\theta)$. Difficult aspects occur as regards to the separation of $h_0$. In the second variant, the pressure distribution law of case infinitely long bearing is applied:

$$p = p_s + \frac{6 \cdot \mu \cdot R^2 \cdot \omega}{c^2} \cdot \frac{\varepsilon \cdot (2 + \varepsilon \cdot \cos(\theta)) \cdot \sin(\theta)}{(2 + \varepsilon^2) \cdot (1 + \varepsilon \cdot \cos(\theta))^2}$$  \hfill (6)

Expression (6) is recalculated through replacement of $h_0$ given by relation [1, 2]:

$$h_0 = \frac{2 \cdot c \cdot (1 - \varepsilon^2)}{2 + \varepsilon^2}$$  \hfill (7)

with expression $h_0$ [9]:

$$h_0 = c - 2 \cdot \frac{L \cdot c}{\pi}$$  \hfill (8)

and obtain:

$$p = p_s + \frac{6 \cdot \mu \cdot R \cdot \omega}{c^2 \cdot \pi} \cdot \frac{(\pi \cdot R - 2 \cdot L \cdot \varepsilon)}{(1 - \varepsilon^2)} \cdot \frac{\varepsilon \cdot (2 + \varepsilon \cdot \cos(\theta)) \cdot \sin(\theta)}{(1 + \varepsilon \cdot \cos(\theta))^2}$$  \hfill (9)

According to the proposed rationale [8] pressure increase $p - p_s$ is corrected, with function $k(\theta)$ which leads to the pressure law of the case under observation:

$$p = p_s + \frac{6 \cdot \mu \cdot R \cdot \omega}{c^2 \cdot \pi} \cdot \frac{(\pi \cdot R - 2 \cdot L \cdot \varepsilon)}{(1 - \varepsilon^2)} \cdot \frac{\varepsilon \cdot (2 + \varepsilon \cdot \cos(\theta)) \cdot \sin(\theta)}{(1 + \varepsilon \cdot \cos(\theta))^2} \cdot k(\theta)$$  \hfill (10)

### 4. ASPECTS OF THE BEHAVIOR OF PROPOSED EQUATION

The journal bearing's loads, according to the model proposed by this paper are calculated with the help of the new pressures equation. Experimental investigation on thermal effects in journal bearings [5], reveals that the cyclic variation of the shaft temperature in the circumferential direction is small and that the shaft can be considered as an isothermal component. Thus in relation (4), the medium lubricant viscosity corresponding to the shaft temperature $T_s$ is taken into account.

The analytical calculation of the main parameters of bearings is made by means of the classic equations provided by the literature in the field and of the distribution law yielded by the expression (10). Then, the calculation of loads, flows, friction coefficients, maximum pressure in the hydrodynamic film are done by means of the classic relations presented in the paper [2].

Table 1 presents the experimental data and, respectively, analytical calculations for loads in a number of concrete situations to be found in the literature [10, 11].

In all cases under analysis, the bearings are single-groove, steadily loaded, and the flow is laminar.

A good agreement between theory and experiment is noticed, in the case of small eccentricity ratios, while with increased eccentricity ratios errors are unacceptable.
### Table 1

<table>
<thead>
<tr>
<th>experimental data source</th>
<th>$T_{\text{shaft}}$ [°C]</th>
<th>$\omega_{\text{shaft}}$ [rad/sec]</th>
<th>$W_{\text{experimental}}$ [N]</th>
<th>$W_{\text{calculated through relation (10)}}$ [N]</th>
<th>% $W$, Error theory-experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lund $\varepsilon = 0.275$</td>
<td>68.3</td>
<td>366.40</td>
<td>1100</td>
<td>1156</td>
<td>+5.11%</td>
</tr>
<tr>
<td>Lund $\varepsilon = 0.300$</td>
<td>67.5</td>
<td>366.40</td>
<td>1500</td>
<td>1560</td>
<td>+4.33%</td>
</tr>
<tr>
<td>Lund $\varepsilon = 0.350$</td>
<td>66.1</td>
<td>366.40</td>
<td>2250</td>
<td>2340</td>
<td>+4.23%</td>
</tr>
<tr>
<td>Dowson $\varepsilon = 0.254$</td>
<td>52.0</td>
<td>157.08</td>
<td>2250</td>
<td>2367</td>
<td>+5.21%</td>
</tr>
<tr>
<td>Dowson $\varepsilon = 0.574$</td>
<td>47.5</td>
<td>157.08</td>
<td>11000</td>
<td>16474</td>
<td>+48.91%</td>
</tr>
</tbody>
</table>

### 5. CONCLUSIONS

With the help of the distribution pressure law one can calculate the finite bearings’ parameters with small loads. Use of the the second variant presented in section 3 is not difficult at all, and the obtained results justify the rationale regarding the correction of the pressure field.

The new mathematical model thus expands calculation accuracy over to small loads, completing the domain of medium and large eccentricity ratios covered by equation (1).

### 6. REFERENCES