CONSIDERATIONS ON THE SHAVERS RELIABILITY
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Key words: reliability, gears, shavers

Abstract: As tools meant for finishing gears, the shavers provide high processing precision and an extremely high productivity, being some extremely expensive tools; therefore, the problems connected with the shavers feasibility, both in the design and in the processing and exploitation stages, are particularly up-to-date and extremely interesting. To achieve these goals it is recommended that the following conditions should be fulfilled even since the design stage: the diameter of the shaver should be as large as possible, the positive displacement of the profile of the new shaver teeth, the wear of the side on the height of the shaver tooth should be even, and the shaver as wide as possible.

The value of the technical products is given by their functional technical, economical, reliability and aesthetic parameters. Reliability $F$ (operating security – SF), in the broader sense of general systems theory, represents the property (capacity) of a product to maintain the output parameters within acceptable limits, in the given operating conditions. The characteristic of a product (tool) to be able to function at a given time is called availability.

Reliability can be defined from two perspectives: qualitative and quantitative.

Qualitatively, reliability can be defined as the capacity of a product to perform the function it was meant for, in the given conditions, over a prescribed period of time;

Quantitatively, it is the characteristic of a product expressed by the probability of fulfilling the imposed function over a given period of time, and under certain operating conditions.

The quality level of reliability of the shavers depends on: the calculus methods used for their design (reliability forecast) which can help to make measurements based on preliminary data, processing technology; methods and means used to control sharpening and re-sharpening the shavers; the exploitation conditions that correspond to certain service requests.

Mathematically, reliability is a probability of a random variable, time or number of functioning cycles (operating life).

If we know the moment when the shaver was put into operation and if we assume that, at $t = 0$, there are $N_0$ shavers in operating condition, then, at some point between the period $(t, t + \Delta t)$, $N$ shavers are still operating.

Therefore, during $\Delta t$ it is considered $\Delta N = N_0 - N$ faulty shavers.

If $(-\lambda)$ is the constant factor of proportionality we can admit that:

$$\Delta N = (-\lambda) \cdot N \cdot \Delta t$$

(1)

If $\lambda > 0$ and $\lim_{\Delta t \to 0} \frac{\Delta N}{N} = \frac{dN}{dt}$, then the relation (1) becomes:

$$\frac{dN}{dt} = -\lambda \cdot N$$

(2)

From differential equation (2) with separable variables, we obtain:

$$\ln N = -\lambda t + C$$

$$N = C \cdot e^{-\lambda t}$$

(3)

From boundary conditions, when $t = 0$, $N = N_0$, we obtain:
\[
\frac{N}{N_0} = e^{-\lambda t}
\]  

(4)

If \( R = \frac{N}{N_0} \), the proportion of operating shavers at the moment \( t \), the mathematical expression of reliability is:

\[
R(t) = e^{-\lambda t}
\]  

(5)

The relation (5) expresses the possibility that the shaver to function with no flaws within the period \((0, t)\) under specified conditions.

If \( \lambda \) is variable in time, we can write:

\[
\frac{dN}{dt} = -N \cdot \lambda(t)
\]  

(6)

and then:

\[
R(t) = \frac{N}{N_0} = e^{-\int_{\lambda(t)} t dt}
\]  

(7)

The relation (7) can also be called survival function or the function of operating security, of exponential type with the extreme values \( R(t) = 1 \) and \( R(\infty) = 0 \).

When we know the initial moment of operating the shaver, we consider reliability within a period of time \((t_0, t_f)\), that is conditional reliability.

If we know the function \( R(t) \), then the additional function

\[
F(t) = 1 - R(t)
\]  

(8)

is called function of breakdown distribution while the frequency density of breakdowns is:

\[
f(t) = \frac{dF}{dt} = -\frac{dR}{dt} = \lambda \cdot e^{-t}
\]  

(9)

With the above clarifications, we can define the rate of breakdowns:

\[
\lambda(t) = -\frac{\frac{dR}{dt}}{R} = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{R(t)}
\]  

(10)

Therefore the expression of reliability is:

\[
R(t) = \frac{f(t)}{\lambda(t)}
\]  

(11)

where:

\[
\lambda(t) = \lim_{\Delta t \to 0} \left[ \frac{\text{Probability that the tool used during interval } (0, \Delta t) \text{ to breakdown in interval } (t, t + \Delta t)}{\Delta t} \right]
\]  

(12)

The mean value of good functioning time (MTBF) is:

\[
MTBF = M(t) = \mu = \int_0^\infty f(t) dt = \int_0^\infty R(t) dt
\]  

(13)

In the theory and practice of shavers reliability, the following statistical models are used:

1. **Exponential model**, in which the main parameters are:
   - **Probability density**: \( f(t) = \lambda \cdot e^{-\lambda t} \);
   - **Rate of breakdowns**: \( \lambda = ct > 0 \);
   - **Distribution function**: \( F(t) = 1 - e^{-\lambda t} \).
For \( \lambda \cdot t < 0.01 \), the following relations may be acceptable roughly:

\[
F(t) = e^{-\lambda t}; \quad R(t) = e^{-\lambda t}; \quad MTBF = \frac{1}{\lambda}; \quad \sigma^2 = \frac{1}{\lambda^2}
\]

where \( \sigma \) is dispersion.

Thus, reliability estimation is done using the test “n” from “n” or the test “r” from “n”.

- **Weibull Model**, in which:

\[
\lambda(t) = \frac{\beta}{\eta} \cdot (t - \gamma)^{\beta-1}
\]

where \( \eta, \beta \) si \( \gamma \) are Weibull distribution parameters namely: \( \beta \) is the parameter of reliability function determining the shape of distribution curve; \( \eta \) is scale parameter; \( \gamma \) is the location parameter, which determines the position of the variation curves in relation to the time horizontal:

\[
f(t) = \frac{\beta}{\eta} \cdot \left(\frac{t - \gamma}{\eta}\right)^{\beta-1} \cdot e^{-\left(\frac{t - \gamma}{\eta}\right)^\beta};
\]

\[
R(t) = e^{-\left(\frac{t - \gamma}{\eta}\right)^\beta};
\]

\[
F(t) = 1 - e^{-\left(\frac{t - \gamma}{\eta}\right)^\beta};
\]

\[
MTBF = \eta \cdot \Gamma\left(1 + \frac{1}{\beta}\right);
\]

\[
\sigma^2 = \eta^2 \cdot \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \Gamma^2\left(1 + \frac{1}{\beta}\right)\right],
\]

where \( \Gamma \) is Euler’s integral for the Gamma function, whose values are indicated in statistical tables in the specialty literature.

- **Gamma Model**, in which the random variable follows a gamma distribution parameter \( \alpha (\alpha \in N \) and represents the failure rate), if the density distribution is given by:

\[
f(t) = \begin{cases} 
\frac{1}{\Gamma(\alpha)} \cdot t^{\alpha-1} \cdot e^{-t}, & \text{daca } t > 0, \\
0, & \text{daca } t \geq 0.
\end{cases}
\]

- **Rayleigh Model** is commonly used in the study of shavers wear; it is a particular case of Weibull distribution for \( \beta = 2 \); the model parameters are:

\[
F(t, \theta) = 1 - e^{-t^2/\theta}; \quad t \geq 0, \quad \theta > 0;
\]

\[
\lambda(t) = \frac{f(t, \theta)}{1 - F(t, \theta)};
\]

\[
f(t, \theta) = 2 \cdot \theta^{-1} \cdot t \cdot e^{-t^2/\theta}; \quad t \geq 0, \quad \theta > 0;
\]

\[
\sigma^2 \approx 0.463 \cdot \theta,
\]

where: \( \theta \) is the average durability (MTBF).

- **Alpha Model**, which has the following distribution function:
\[
F(t, \alpha, \beta) = \frac{1}{\sqrt{2 \cdot \pi}} \cdot \Phi(\alpha) \cdot \int_{\beta - \alpha}^{\infty} e^{-\frac{z^2}{2}} dz = \frac{1 - \Phi\left(\frac{\beta - \alpha}{t}\right)}{\Phi(\alpha)},
\]

where \( \Phi(\mu) = \frac{1}{\sqrt{2 \cdot \pi}} \cdot \int_{-\infty}^{\mu} e^{-\frac{z^2}{2}} dt \), \( iar \, z = \frac{\beta}{u} - \alpha \).

The above considerations are generally valid for the reliability study of any cutting tools; different models of statistic distribution have been established during the sustainability study of different tools.

In the case of the study of shaver sustainability, we consider the fact that its failure is less due to the tear and more to the exceeded allowable wear. The shaver operating life is a function that depends on many variables, within which it is advisable to complete as many good operation cycles as possible with as big as possible MTBF.

After examining the formation method of reliability establishing models, we can draw some conclusions and make some recommendations for maximizing the shaver durability.

Firstly, it is recommended that the shaver diameter to be as large as possible, this increasing the cutting speed and the operation productivity. The head of the shaver tooth has to leave the toothing freely, not to scrape anything else but the active side of the processing wheel (to avoid the interference of the combined profiles, i.e. shaver - processing wheel); this thing can be done particularly by shortening the head of the shaver tooth.

Secondly, so as the number or possible re-sharpening to be as high as possible, it is recommended to have a positive displacement of the new shaver teeth profile; by re-sharpening, the nature of displacement is modified, becoming negative for the extremely worn shaver. The maximum coefficient of positive displacement of the shaver profile in transversal section is determined as follows:

\[
S_{t, noa} = \frac{\pi \cdot m_n}{2} + 2 \cdot \Delta S_{t, noa} = \frac{\pi \cdot m_n}{2} + 3 \cdot m_n \cdot \xi_t \cdot tg \alpha_0,
\]

\[
\xi_t = \frac{\Delta S_{t, noa} \cdot \cos^2 \beta}{m_n \cdot tg \alpha_0},
\]

which in normal section has the expression:

\[
\xi_n = \frac{\Delta S_{n, noa}}{m_n \cdot tg \alpha_0},
\]

\( \Delta S_{n, noa} \) is considered on the dividing circle and can be checked through the length over \( N \) teeth.

Thirdly, so as MTBF to be as high as possible, it is desirable that the wear of the side of the height of the shaver tooth to be even. The uneven wear is determined by the cutting force and speed and by the functional geometry, which are not constant on the tooth height; it is also determined by the evenness of the height of cutting channels.

Fourthly, the shaver life duration and quality of the shaver operation are positively influenced by the shaver width which is as large as possible (it is recommended that \( B_s = 20mm \) for \( d_s < 190mm \) and \( B_s = 25mm \) for \( d_s > 190mm \)). In contrast, the volume and, respectively, the shaver tooth mass influence the durability and quality of the shaving operation; the bigger the tooth mass, the smaller the wear, the lower the deformation and, as a consequence, the cutting quality is higher; the bigger the shaver mass, the smaller the temperature rise, and so, the thermal stress is lower and the wear is more insignificant.
References: