PROJECT RESOURCE PLANNING WITH AVAILABLE RESOURCES

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Abstract—A complex project, as well as in the construction industry, requires the engagement of more than one category Work resources use more types of resource-type material and the disposal of the corresponding budget Work resources are usually considered the available amount of time. Resources type of material is characterized by purchases, spending and inventories. By analogy, is considered the budget: security or inflow of money, payment of costs and liquidity. This paper points to the same time planning the following types of resources and brings together two well-known problems: planning of materials and planning of liquidity.

Keywords—construction project, liquidity, optimization, mathematical models.

I. INTRODUCTION

Every project consists of activities that are appropriate logical and technological dependence, using the necessary resources (manpower, machinery, materials, etc.) and require a certain time to perform. For project planning, the standard software for Project Management (PM) (in our Primavera and MS Project). Resources are usually available in limited quantities. And when there are enough resources, they must be used rationally. The projects in the construction industry is characterized by more specialist workers needed machinery, a large number of different types of materials and high costs.

Here we consider three types of resources from the point of their consumption of the activity: Work type (number of units of resources on the day of the activity), type of material (the number of unit resources for activity) and money (the number of units of the resource unit).

Many authors detail all aspects of the use of those types of resources. Especially important are the costs for the first two types of resources [1]-[8].

Software Primavera and MS Project support the planning of the project with available resources type Work. It is understood that provision of the necessary resources-type material and necessary budget. However, software CA-SuperProject has since 1990 supported the recording of procurement, budget and inventory leveling resource type material as incurred negative stock.

1) Are not achieved the expectations of users to Primavera and MS Project provide more convenient program planning resource material.

2) As a result, some authors have presented an interactive process application software for PM with a view to planning materials [9]-[16].

Two known problems, planning materials and planning of liquidity on the project, will be below united and resolved as a problem.

II. TIME AND COST OF THE PROJECT

Let certain project make m types of work or activities marked with Ai. For their performance is engaged n resource category Bj type Work (workers, machinery, materials, etc.) and require a certain time to perform. For project planning, the standard software for Project Management (PM) (in our Primavera and MS Project). Resources are usually available in limited quantities. And when there are enough resources, they must be used rationally. The projects in the construction industry is characterized by more specialist workers needed machinery, a large number of different types of materials and high costs.

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the first type resources and the amount of other types of resources. Planner or analyst defines the most appropriate resource intensity $B_j$ type Work for activity $A_i$, as the number of units of resources $u_{ij}$ per day. For example, the number of employees specified a category. On the basis of hours required $W_{ij}$, intensity $u_{ij}$ and daily working hours of resources $B_j$, are calculated the required time engagement of each category of these resources:

$$t_{ij} = \frac{W_{ij}}{(a_{ij} + a_{ij}^+) \cdot u_{ij}}, \quad j \in J_i, \quad i \in I$$

(1)

where: $a_{ij}$ - the duration of the normal working hours, and $a_{ij}^+$ - overtime work for $B_j$.

If activity uses only one category of resources, time $t_i$ for the duration of activity as the time of engagement resources. If activity uses several categories of resources, $t_i$ is equal to the time resources that are no longer engaged in its execution.

$$t_i = \begin{cases} t_{ij}, & n_i = 1, \quad j \in J_i \\ \max_{j \in J_i} t_{ij}, & n_i > 1 \end{cases}, \quad i \in I$$

(2)

The total amount $M_{ik}$ of material units $D_k$ for activity $A_i$ is allocated to its days of execution $t_i$ in several ways: linear (determined daily needs $u_{ik} = M_{ik} \cdot t_i$, $k \in K, \quad i \in I$), or application of certain rules (Work Conture) that supports the software. The scheduler can define their own rules.

Interdependence of activities and their timing is determined the minimum duration of the project $T_{p_{min}}$.

It can be increased on conditional minimum duration $T_{p^*}$, if it does not have sufficient resources, $T_{p^*} > T_{p_{min}}$.

Each resource has its own prices to determine the costs for the activity in which it is used. There are variable costs for each unit of resources and separate fixed costs. Resources $B_j$ define the unit prices given monetary units for 1 hour in normal working hours and higher unit costs for overtime work. Resources $D_k$ have a price per unit of resources, regardless of the form of working time resources $B_j$. With the above values are formed variable costs per unit of resources. Fixed costs can be defined for each schedule resources $B_j$ (apply to all activities that use the same, regardless of the number of units of resources in individual schedules). Resources $D_k$ correspond to possible fixed costs for the total amount of the activities (regardless of the number of schedules and quantities). All types of costs can be constant during the project implementation or different for certain periods of the project. The same resource can have a different price (e.g. MS Project supports five price list), with one cost with a corresponding set of activities and another cost on a new set of activities.

The sum of the cost of appropriate resources to provide the cost of activities to be added to group activities, higher levels of the project and the project as a whole. At the same time, can be set up new types of fixed costs for the activity, any level of the project and the project as a whole. Costs can be seen in appropriate time units.

$$c_i = \sum_{j \in J_i} c_{ij} + \sum_{k \in K} c_{ik}$$

(3.1)

$$C = \sum_{i \in I} c_i$$

(3.2)

$$C(t) = \sum_{i \in I} c_i(t), \quad t \in T$$

(3.3)

where: $c_{ij}$ and $c_{ik}$ are resource costs $B_j$ and $D_k$ for $A_i$, respectively, $c_i$ total costs for $A_i$, $C$ total project costs, $c_i(t)$ and $C(t)$ costs for $A_i$ and project in the observed time unit $t$.

Remark 1: Resources $D_k$ of type Material have several forms of expenses (see Hendrickson, 2008, Chapter 4.8 Inventory Control): (1) procurement, (2) administration, (3) keeping the stock, and (4) emergency purchases due to lack of materials. These expenses are not considered in more detail, but not complex to be built in the displayed models and algorithms.

Software PM reviews the project costs and the required elements of the project (activities, group activities, resources), time periods (day, week, month, etc.). Furthermore, the software supports three types of scheduling resource costs to activities 'Accrue At': 'Start' (total cost at the beginning of activities), 'Prorate' (according to the engagement of resources), and 'End' (total cost at the end of activities).

The sum of needs for a resource $B_j$ type Work (4.1) and $D_k$ type Material (4.2) in all activities $A_i$, which are performed on an observed day $t$, give the total required amount of the appropriate resources to the project:

$$u_{ij}(t) = \sum_i u_{ij}(t), \quad j \in J, \quad t \in T$$

(4.1)

$$u_{ik}(t) = \sum_i u_{ik}(t), \quad k \in K, \quad t \in T$$

(4.2)

where: $u_{ij}(t)$ and $u_{ik}(t)$ are total needs $B_j$ and $D_k$, respectively, $u_{ij}(t) > 0$ and $u_{ik}(t) > 0$ if $A_i$ performed in $t$, $u_{ij}(t) = 0$ and $u_{ik}(t) = 0$ is not performed in $t$, $T$ a set of days for the period of the project.
Fig. 1. shows the life-cycle of an ERP software that is used to support project management.

The daily required amount of resources for all activities cannot be higher than the daily amount available for the project in its entirety. If there are exceeding available resources, should be reduced to the use of available resources in the border.

Resources $B_j$ type Work define available daily amount $u_j(0(t))$. Max Units: Numbers of workers of certain specialties, or numbers of certain types of machinery, $j \in J$, $t \in T$. These amounts must ensure that the performance of at least one activity of the day $t$, or cannot be less than the needs of $A_i$ with a maximum daily commitment observed $B_j$ when $A_i$ done in the most serendipitous period of the project, which provides the shortest duration of the project (5.1). Max Units may be constant values $u_j(0(t)) = u_j$ in the course of the project or different constant values for certain periods of the project. Total daily resources needs $B_j$ cannot be larger than the available quantity (5.2).

\[
\begin{align*}
    u_j(0(t)) & \geq \max_{i \in I} u_{ij}(t), \quad j \in J, \ t \in T \\
\sum_{i \in I} u_{ij}(t) & \leq u_j(0(t)), \quad j \in J, \ t \in T
\end{align*}
\]

If the needs are greater than the available quantity, PM software performs resource leveling $B_j$ type Work, or the requirement to be available in the resource limit (allows to carry out priority activities while having the available resources and the residual activity shifted to periods with sufficient resources). Time can be increased above a certain minimum duration of the project calculated by observing only depending on time and activities (considering that there is enough resources type Work).

Total daily needs of resource-type material calculated to plan the project (schedule of activities at the time) with resources type Work. On Fig. 2. is given a generic model of resource planning with an organization.
III. PROBLEMS OF RESOURCES PLANNING

Basic aspects of planning 'non-spendable' Work resources are at the front. The other two types of resources, material and money for expenses, have identical characteristics 'spendable' resource: input, output, and status (availability).

Resources type material and money must be provided in the total amount required for the project. Or are substantial and their available quantities in the days when they are used.

\[ U_k^+ = \sum_{t \in T_k} u_k^+ = M_k, \quad k \in K \]  \hspace{1cm} (6.1)

\[ U_k^{cum}(t) = \sum_{i=1}^{\tau} \sum_{t \leq \tau} u_{ik}(\tau), \quad k \in K, \quad t \in T \]  \hspace{1cm} (6.2)

\[ U_k^{+cum}(t) = \sum_{t < \tau} u_k^+, \quad k \in K, \quad t \in T \]  \hspace{1cm} (6.3)

\[ S_k(t) = U_k^{+cum}(t) - U_k^{cum}(t), \quad k \in K, \quad t \in T \]  \hspace{1cm} (6.4)

\[ C^+ = \sum_{t \in T} c^+(t) = S \]  \hspace{1cm} (7.1)

\[ C^{cum}(t) = \sum_{i=1}^{\tau} \sum_{t \leq \tau} c_i(t), \quad t \in T \]  \hspace{1cm} (7.2)

\[ C^{+cum}(t) = \sum_{t \leq \tau} c_i^+(t), \quad t \in T \]  \hspace{1cm} (7.3)

\[ L(t) = C^{+cum}(t) - C^{cum}(t), \quad t \in T \]  \hspace{1cm} (7.4)

where: \( t_{ks}^+ \) and \( T_k^+ \) terms for procurement \( D_k \) and a set of these terms, \( t_{ks}^+ \in T_k^+ \), \( u_{ks}^+ \) and \( U_k^+ \) the quantities purchased in all periods \( t_{ks}^+ \) and total purchases \( D_k \).

\[ U_k^{cum}(t) \] cumulative need \( D_k \) since the beginning of the project as of the time unit \( t \), \( U_k^{+cum}(t) \) cumulative purchases ending with \( t \), \( S_k(t) \) condition or stocks \( D_k \) in \( t \), \( t_{\alpha}^+ \) and \( T^+ \) terms of cash flow and a set of such terms, \( t_{\beta}^- \) and \( T^- \) terms of payment of costs (outflows) and a set of such terms, \( t_{\beta}^+ \) cumulative cost of incoming payments ending with \( t \), \( C^{+cum}(t) \) cumulative cash flow ending with \( t \), \( L(t) \) cash position or liquidity in \( t \).

They can be seen following numbers and matching sets of indices: \( p_s^+ \) terms \( t_{ks}^+ \) procurement of resources \( D_k \), \( q^+ \) terms \( t_{\alpha}^+ \) cash inflow, \( q^- \) terms \( t_{\beta}^- \) payment of costs, \( P_s^+ = \{1, \ldots, p_s^+\} \) index set \( \alpha \) for \( t_{ks}^+ \), \( s \in P_s^+ \), \( Q^+ = \{1, \ldots, q^+\} \) set \( \alpha \) for \( t_{\alpha}^+ \), \( \alpha \in Q^+ \), and \( Q^- = \{1, \ldots, q^-\} \) set \( \beta \) for \( t_{\beta}^- \), \( \beta \in Q^- \).

**Position 1.1.** Stocks \( S_k(t) \) resources \( D_k \) of type Material in the term \( t \) are equal to the difference between cumulative and cumulative consumption of
procurement, from the beginning of the project up to that term (6.4). Position 1.2. $S_k(t)$ cannot be less than the need $u_{ij}(t)$ of at least one activity $A_j$ which is carried out in the $t$. In general, the required negative stock (8.1). If there are negative stock (8.2), it is necessary to allow the carrying out of priority $A_i$ with available $D_k$, move the other activities in the coming period with sufficient $D_k$ and level resources $B_j$ of type Work, $j \in J$.

$$S_k(t) \geq 0, \ k \in K, \ t \in T$$

(8.1)

$$S_k(t) < 0, \ k \in K, \ t \in T$$

(8.2)

Position 2.1. The balance of cash or liquidity $L(t)$ in term $t$ is the cumulative difference between inflows and outflows cumulative from the beginning of the project up to that term (7.4). Position 2.2. Strict rule that liquidity in each day on project (7.4) must be negative, in practice replaces the usually acceptable requirements: Liquidity $L(t_p^\beta)$ in term $t_p^\beta$ payment of expenses cannot be less than the cost $c_i(t_p^\beta)$ of derivative works (engaged $u_{ij}$ and $u_{ik}$) of at least one activity $A_j$, which is done in the period since the previous term of payment of costs $t_p^\beta - 1$ ending with the observed $t_p^\beta$.

It can be adapted to carry out the payment of costs at the end of each month and at the end of the project for the last month. It follows that the required non-negative liquidity (9.1). If there is negative liquidity (9.2), it is necessary to allow the carrying out of priority $A_i$ with available money to pay its costs $c_i(t_p^\beta)$, move the other activities in the coming period with enough money, and resources leveling $B_j$ type Work, $j \in J$.

$$L(t_p^\beta) \geq 0, \ t_p^+ \in T^+$$

(9.1)

$$L(t_p^\beta) < 0, \ t_p^+ \in T^+$$

(9.2)

IV. VARIANTS OF THE PROBLEM

Four variants of the problem, and combinations thereof are formed in accordance with the two groups of conditions: (a) the terms and a number of purchases, $t_k^+$ and $u_k^+$, resource $D_k$ type Material, and (b) for the terms and the amount of cash flow, $t_{k\alpha}^+$, $c_{\alpha}^+$.

Problem 1. Formed acceptable minimum duration of the project plan $T_p^*$ with available quantities $u_{j0}(t)$ resource $B_j$ of type Work, $j \in J, \ t \in T^*$.

Determine parameters $t_{k\alpha}^+, \ u_{k\alpha}^+, \ t_{\alpha}^+$ and $c_{\alpha}^+$ which does not change $T_p^*$.

General mathematical model:

$$\text{min} \text{ } T_p(t) = T_p$$

(10)

$$x_i \geq 0, \ i \in I \land R_i^- = \emptyset$$

(11)

$$x_i \geq y_{i\lambda}, \ i \in I \land R_{i\lambda}^- \neq \emptyset$$

(12.1)

$$y_{i\lambda} = x_{i\lambda} + t_{i\lambda}, \ \lambda \in R_{i\lambda}^- \neq \emptyset, \ i \in I$$

(12.2)

$$y_i = x_i + t_i, \ i \in I$$

(13)

$$y_i \leq T_p, \ i \in I \land R_i^+ = \emptyset$$

(14.1)

$$T_p = T_p^*$$

(14.2)

$$\sum_{i \in I} u_{ij}(t) \leq u_{j0}(t), \ j \in J, \ t \in T$$

(15)

$$S_k(t) \geq 0, \ k \in K, \ t \in T$$

(16)

$$L(t_p^\beta) \geq 0, \ t_p^\beta \in T^-$$

(17)

$$x_i, y_i \geq 0, \ i \in I, \ T_p \geq 0$$

(18.1)

$$x_{i\lambda}, y_{i\lambda} \geq 0, \ \lambda \in R_{i\lambda}^- \neq \emptyset, \ i \in I$$

(18.2)

where: $x_i$ is the beginning of activities $A_j$, $i \in I$, $y_i$ completion $A_j$, $R_{i\lambda}^-$ index set $\lambda$ predecessor, activities $A_{i\lambda}$ of which depends $A_j$, $\lambda \in R_{i\lambda}^- \neq \emptyset$, $R_i^+$ index set of activities that depend on $A_j$, $T_p$ the duration of the project with the conditions (11)-(14.1) and (15)-(18.2), $T_p^*$ the minimum duration of the project with conditions (15) and without conditions (16,17).

A mathematical model of linear programming (LP) with absolute time units is defined $t = 1,...,T_p$. Unknown quantities can be monitored in (18.1) and (18.2). The model describes the following problems and 3-4, with appropriate restrictions on the interpretation of (14.2), as well as some of the elements in (16) and (17).

The function criteria (10) minimizes the duration of the project, constraints (11) determines the start of the initial activity $A_j$ that is independent of other activities $A_j$ has $R_{i\lambda}^- = \emptyset$, Constraints (11) provides the start of initial activity $A_j$ that do not depends on other activities $A_j$ has $R_{i\lambda}^- = \emptyset$.

Equation (12.1) defines the beginning $A_j$ with dependencies (has $R_{i\lambda}^- \neq \emptyset$) can not be earlier than the end of each of its predecessor $A_{i\lambda}$, $\lambda \in R_{i\lambda}^-$, (12.2) calculates the completion of each $A_{i\lambda}$ (start plus duration) for application in (12.1), (13) forms the completion of
each $A_i$, (14.1) stipulates that the duration of the project $T_p$, or completion of the project can not be earlier than the end of each final $A_i$ without successor (no further activities, has $R_i^+ = \emptyset$), (14.2) stipulates that $T_p$ keeps the initial minimum duration $T_p^{\#}$, specific requirements (5.2) and repeated by (15), which defines that the use of each resource $B_j$ type Work does not exceed the available quantities, conditions (16) are derived from (6.4) and determine the nonnegative resource stocks $D_k$ type Material, (17) repeats (9.1) for the determination of the earliest completion of the non-negative liquidity (18.1) and (18.2) are the natural conditions of non-negativity of variables. Unknown to the beginnings and endings of action are always the earliest time, but not later than the time, from the analysis of time to the project. Model easily solved without conditions (15) - (17) with software for LP and complex involvement (17). Conditions (16) and (17) contain other unknown quantities from (6.1) - (7.4). As a result, simpler is an interactive software application for PM, defined by the following algorithms. Position 2.1. Linear conditions (12.1) are replaced by non-linear definition (19) for the earliest start of each $A_i$ which depends on several $A_{i_\lambda}$ (the earliest start off $x_{i_\lambda}^E$ for $A_{i_\lambda}$ equal to the earliest completion $y_{i_\lambda}^E$ of $A_{i_\lambda}$ which ends last). Position 2.2. Linear conditions (14.1) are replaced by non-linear condition (20) for the completion of the project (duration), and the earliest and latest completion of the project $T_p$ equal to the earliest completion $y_{i_\lambda}^E$ for $A_{i_\lambda}$ from a set of activities with no successor being the last ends):

$$x_{i_\lambda}^E = \max_{\lambda \in R_i^-} y_{i_\lambda}^E, \quad i \in I \land R_i^- \neq \emptyset$$  \hfill (19)

$$T_p = \max_{i} x_{i_\lambda}^E, \quad i \in I \land R_i^+ = \emptyset$$  \hfill (20)

V. CONCLUSION

The paper has exposed the planning of the project with available resources type Work, introducing two plans: (1) procurement of resources type Material to carry out activities, and (2) providing money for achieving liquidity in terms of payment of costs. Defines general mathematical models for the four problems. The proposed algorithms of their solution based on interactive software application standard for PM that supports leveling the resources type Work.

To issues of importance for projects in the construction industry and can be applied in other areas of business. Future research should include a detailed discussion of the cost of materials and implementation of the project.

For materials are essential forms of the following costs: (1) procurement, (2) administration, (3) storage of inventory, (4) of any emergency purchases due to lack of material with regular procurement, and (5) the corresponding fixed costs. In the project with a longer duration may be present that many elements of the plan have multiple changes in the weather. For example: (i) changes in the cost, (ii) delays activities, (iii) changes in the amount of cash flow in the planned terms, (iv) changes in the term while maintaining the planned amount of cash flow, and (v) changes in the terms and the amount of cash flow.

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